IS GRAPHENE ON THE EDGE OF BEING A TOPOLOGICAL INSULATOR?

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DYNAMICAL SYMMETRY BREAKING IN GRAPHENE

We address the question of dynamical breakdown of symmetry in graphene many-body theory

\[ H = -i e F \int d^2r \ \nabla \cdot \Psi(r) + e^2 \int d^2 r_1 \int d^2 r_2 \ \rho(r_1) \frac{1}{|r_1 - r_2|} \rho(r_2) \]

A gap can be opened in the spectrum through the generation of different mass terms, characterized by the symmetry broken in each case:

- parity-invariant mass term (G. W. Semenoff, PRL 53, 2449 (1984)):

\[ H = m_0 \int d^2r \ \overline{\Psi}(r) \Psi(r) = m_0 \int d^2r \ \Psi^+(r) (\sigma_3 \otimes \tau_3) \Psi(r) \]

breaks chiral symmetry \( CS : \Psi(r) \rightarrow (1 \otimes \tau_1) \Psi(r) \)

- Haldane mass term (F. D. M. Haldane, PRL 61, 2015 (1988)):

\[ H = m_H \int d^2r \ \overline{\Psi}(r) \gamma_5 \Psi(r) = m_H \int d^2r \ \Psi^+(r) (\sigma_3 \otimes 1) \Psi(r) \]

breaks invariance under parity \( P_y : \Psi(x, y) \rightarrow (\sigma_1 \otimes \tau_1) \Psi(x, -y) \)

and it also breaks time-reversal symmetry \( T = KP \), with

\[ P : \Psi \rightarrow (\sigma_2 \otimes \tau_2) \Psi \]
DYNAMICAL MASS GENERATION IN GRAPHENE

There have been many analyses of the dynamical generation of a parity-invariant mass:

- **Gap equation, 1/N approximation.** D. V. Khveshchenko, PRL 87, 246802 (2001); E. V. Gorbar, V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, PRB 66, 045108 (2002).
- **Lattice field theory.** J. E. Drut and T. A. Lähde, PRL 102, 026802 (2009); PRB 79, 241405(R) (2009) ⇒ critical $\alpha_c \approx 1.08$
  (see also W. Armour, S. Hands, C. Strouthos, PRB 81, 125105 (2010))
- **Ladder approximation, static polarization.** J. Wang, H. A. Fertig and G. Murthy, PRL 104, 186401 (2010); O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 80, 165429 (2009) ⇒ critical $\alpha_c \approx 1.62$
- **Gap equation, dynamical screening.** O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 81, 075429 (2010) ⇒ critical $\alpha_c \approx 0.92$
- **Effect of Fermi velocity renormalization.** D. V. Khveshchenko, JPCM 21, 075303 (2009); J. Sabio, F. Sols and F. Guinea, PRB 82, 121413(R) (2010)
- **Ladder approximation, dynamical screening + electron self-energy corrections.** J. G., PRB 85, 085420 (2012) ⇒ critical $\alpha_c \approx 1.75$

But, what about the dynamical generation of a parity-breaking mass in graphene?
DYNAMICAL MASS GENERATION. RENORMALIZATION GROUP

The dynamical mass generation can be characterized by the divergence of the susceptibility

$$\Pi^{(m)}(r) = \left\langle \Psi^+(r) M \Psi(r) \Psi^+(0) M \Psi(0) \right\rangle \sim M \star M$$

A divergence of $\Pi^{(m)}$ at $q \to 0$ will imply the spontaneous development of a condensate

$$\left\langle \Psi^+(r) M \Psi(r) \right\rangle \neq 0$$

The low-energy behavior of the susceptibility is:

- non-interacting theory: $\Pi^{(m)}(q, \omega = 0) \sim |q|$
- interacting theory: $\Pi^{(m)}(q, \omega = 0) \sim |q|^{2\gamma}$

The scaling of $\Pi^{(m)}$ is modified because the interactions introduce dependence on the cutoff $\Lambda$

The independence of $\Pi^{(m)}(q,0) = (Z_M)^{-2} \Pi^{(m)}_{\text{ren}}(q,0)$ on $\mu$ leads to the equation

$$\left( \mu \frac{\partial}{\partial \mu} - 2\gamma \right) \Pi^{(m)}_{\text{ren}}(q,0) = 0 \quad \Rightarrow \quad \Pi^{(m)}_{\text{ren}}(q,0) \sim \mu^{2\gamma}$$

$$\gamma = -\frac{\Lambda}{Z_M} \frac{\partial Z_M}{\partial \Lambda}$$
DYNAMICAL MASS GENERATION. LADDER APPROXIMATION

We can study the divergences of the vertex in the ladder approximation

Screening the interaction in the static RPA, we get the self-consistent equation

\[ \Gamma_M(q = 0; k) = M + ie^2 \mu^\epsilon \int \frac{d\omega_p}{2\pi} \frac{d^{2-\epsilon} p}{(2\pi)^2} \gamma_0 G(p, \omega_p) \Gamma_M(q = 0; p) G(p, \omega_p) \gamma_0 \frac{1}{2\kappa|p - k|} \]

\[ \kappa = 1 + \frac{e^2}{8\nu_F} \]

A suitable way of extracting the scale dependence of \( \Gamma_M \) is to compute the integrals at dimension \( D = 2 - \epsilon \). The only dependence on \( \mu \) comes from the need to introduce a dimensionful \( e^2_0 = \mu^\epsilon e^2 \). The cutoff divergences in \( \Gamma_M \) now appear as poles \( 1/\epsilon, 1/\epsilon^2, \ldots \)

\[ \Gamma_M(q; k) \bigg|_{\text{ren}} = Z_M \Gamma_M(q; k) \quad Z_M = 1 + \sum_{n=1}^{\infty} \frac{d_n(\lambda)}{\epsilon^n} \quad \lambda = \frac{e^2}{4\pi\kappa\nu_F} \]

The anomalous dimension is now

\[ \gamma = \frac{\mu}{Z_M} \frac{\partial Z_M}{\partial \mu} = \frac{\mu}{Z_M} \frac{\partial \lambda}{\partial \mu} \frac{\partial Z_M}{\partial \lambda} = -\lambda d'(\lambda) \]
The anomalous dimension $\gamma = -\lambda \, d_1'(\lambda)$ can be computed perturbatively from the expansion

$$d_1(\lambda) = \sum_{n=1}^{\infty} d_1^{(n)} \lambda^n$$

It turns out that the series for $\gamma$ has a finite radius of convergence

$$\lambda_c \approx 0.456947$$

which coincides precisely with the critical coupling obtained from the gap equation

$$\lambda_c = \frac{8\pi^2}{(\Gamma(1/4))^4}$$

(J. G., JHEP 08, 27 (2012))

(O. V. Gamayun et al., PRB 80, 165429 (2009))

$\lambda_c$ is the critical coupling for the dressed Coulomb potential, and its relation to the nominal coupling $\alpha = e^2/4\pi v_F$ depends on screening effects.

Under static RPA

$$\lambda_c = \frac{\alpha_c}{N\pi} \frac{1}{1 + \frac{N\pi}{8} \alpha_c}$$
The above approach can be improved adding the effect of electron self-energy corrections

\[ \tilde{V}_p(p) = Z_0 v_F + \frac{1}{2\pi|p-k|} \]

\[ \Gamma_M(q = 0; k) = M + ie^2 \mu^2 \int \frac{d\omega_p}{2\pi} \frac{d^{2-q}}{(2\pi)^2} \gamma_0 G(p, \omega_p) \Gamma_M(q = 0; p) G(p, \omega_p) \gamma_0 \frac{1}{2\kappa|p-k|} \]

In this case it makes a difference to choose

\[ M_0 = \gamma_0 \quad \text{or} \quad M_H = i\gamma_1\gamma_2 \]

It turns out that the perturbative series diverges faster for \( M_H \)

We obtain now the critical couplings

\[ \lambda_c \approx 0.5448 \quad \text{for} \quad M_0 = \gamma_0 \]

\[ \lambda_c \approx 0.508 \quad \text{for} \quad M_H = i\gamma_1\gamma_2 \]

which are substantially larger than in the absence of electron self-energy corrections.

(J. G., arXiv:1211.3905)
DYNAMICAL MASS GENERATION AT GENERAL $D$

One can also renormalize the many-body theory at any spatial dimension $D$. Then, chiral symmetry breaking turns out to be the dominant instability at $D = 3$, but there is a crossover in favor of parity breaking at $D = 2$.

The fact that the dynamical breakdown of parity and chiral symmetry breaking do not take place at the same $\lambda_c$ at $D = 2$ can be considered as an anomaly (a consequence of the regularization at $D = 2 - \varepsilon$).

Dimensional regularization is singled out as a method preserving the gauge invariance of the Dirac field theory. But it is worthwhile to look at a different gauge invariant alternative given by the lattice regularization of the Dirac fermions

$$S = \int d^d x \bar{\psi}(r) i(\gamma_\mu \partial_\mu - v_F \gamma \cdot \nabla) \psi(r)$$

$$\Rightarrow \sum_{n, \mu = 0, 1, 2} \eta_\mu(n) \left[ \bar{\psi}(n) U(n + e_\mu) \psi(n + e_\mu) - \bar{\psi}(n) U^+(n) \psi(n - e_\mu) \right]$$
DYNAMICAL MASS GENERATION ON THE LATTICE

In the formulation of staggered fermions, the Dirac fermion components are spread over each unit cell of the lattice, and the mass matrices $M_0$ and $M_H$ are realized in different ways:

$$M_0 = \gamma_0 \quad \Rightarrow \quad S_0 = m_0 \bar{\psi}(n)\psi(n)$$

$$M_H = i \gamma_1 \gamma_2 \quad \Rightarrow \quad S_H = m_H i\bar{\psi}(n)\sum_{ijk} \eta_0 \eta_1 \eta_2 T_i T_j T_k \psi(n)$$

One can compute for instance the susceptibilities

$$\chi_0 = \left. \frac{\partial \langle \bar{\psi}(n)\psi(n) \rangle}{\partial m_0} \right|_{m_0 = 0}$$

$$\chi_H = \left. \frac{\partial \langle i\bar{\psi}(n)\sum_{ijk} \eta_0 \eta_1 \eta_2 T_i T_j T_k \psi(n) \rangle}{\partial m_H} \right|_{m_H = 0}$$

$\chi_0$ and $\chi_H$ remain regular at the transition (consistent with the solution of the gap equation) and show very close critical couplings ($\beta_c \approx 0.05$, $e^2/4\pi v_F \approx 1.6$) for the two symmetry breakings.

$$\beta \equiv \frac{v_F}{e^2}$$
In conclusion:

We have seen that the dynamical breakdown of parity (and time-reversal invariance) may have at least equal strength in graphene as the chiral symmetry breaking.

- in the ladder approximation, electron self-energy corrections increase significantly the critical coupling up to $\alpha_c \approx 2.51$
  (much lower anyhow than that for chiral symmetry breaking, $\alpha_c \approx 3.78$)
- a more comprehensive consideration of different screening effects is achieved with the lattice gauge theory approach, leading to $\alpha_c \approx 1.6$
  (which compares well with the result $\alpha_c \approx 1.75$ from dynamical screening in the RPA)

It is still puzzling that the most sensible values found for $\alpha_c$ would imply the development of a gap in the spectrum of free-standing graphene samples, while there seems to be no experimental observation in that direction
  ⇒ possible development of more exotic effects from the nucleation of loop currents, domains with alternating orientation of the order parameter, etc.