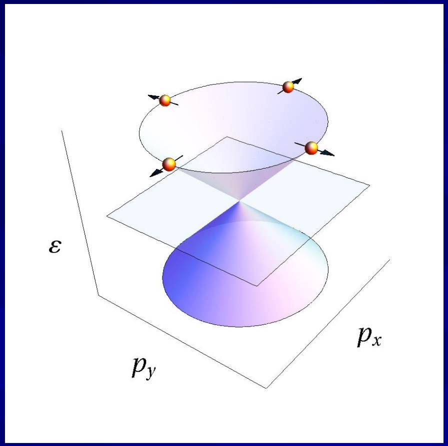
IS GRAPHENE ON THE EDGE OF BEING A TOPOLOGICAL INSULATOR?



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DYNAMICAL SYMMETRY BREAKING IN GRAPHENE

We address the question of dynamical breakdown of symmetry in graphene many-body theory

$$H = -iv_F \int d^2 r \;\; \overline{\Psi}(\mathbf{r}) \;\; \boldsymbol{\gamma}\cdot\partial \;\; \Psi(\mathbf{r}) + \frac{e^2}{8\pi}\int d^2 r_1 \int d^2 r_2 \;\; \rho(\mathbf{r}_1) \; \frac{1}{|\,\mathbf{r}_1 - \mathbf{r}_2\,|} \; \rho(\mathbf{r}_2)$$

A gap can be opened in the spectrum through the generation of different mass terms, characterized by the symmetry broken in each case:

- parity-invariant mass term (G. W. Semenoff, PRL 53, 2449 (1984)):

$$H = m_0 \int d^2 r \;\; \overline{\Psi}(\mathbf{r})\,\Psi(\mathbf{r}) = m_0 \int d^2 r \;\; \Psi^+(\mathbf{r})\,(\sigma_3 \otimes \tau_3)\,\Psi(\mathbf{r})$$

breaks chiral symmetry $\qquad CS : \Psi(\mathbf{r}) \;\to (\mathbf{1}\otimes\tau_1)\,\Psi(\mathbf{r})$

- Haldane mass term (F. D. M. Haldane, PRL 61, 2015 (1988)):

$$H = m_H \int d^2 r \;\; \overline{\Psi}(\mathbf{r})\,\gamma_5 \Psi(\mathbf{r}) = m_H \int d^2 r \;\; \Psi^+(\mathbf{r})\,(\sigma_3 \otimes \mathbf{1})\,\Psi(\mathbf{r})$$

breaks invariance under parity $\qquad P_y : \Psi(x, y) \;\to (\sigma_1 \otimes \tau_1)\,\Psi(x,-y)$

and it also breaks time-reversal symmetry $T = KP$, with

$$P : \Psi \to (\sigma_2 \otimes \tau_2)\,\Psi$$

DYNAMICAL MASS GENERATION IN GRAPHENE

There have been many analyses of the dynamical generation of a parity-invariant mass:

- **Gap equation, 1/*N* approximation**. D. V. Khveshchenko, PRL 87, 246802 (2001); E. V. Gorbar, V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, PRB 66, 045108 (2002).
- **Gross-Neveu interactions**. I. F. Herbut, V. Juricic and O. Vafek, PRB 80, 075432 (2009); V. Juricic, I. F. Herbut and G. W. Semenoff, PRB 80, 081405 (2009).
- **Lattice field theory**. J. E. Drut and T. A. Lähde, PRL 102, 026802 (2009); PRB 79, 241405(R) (2009) $\Rightarrow$ critical $\alpha_c \approx 1.08$

  (see also W. Armour, S. Hands, C. Strouthos, PRB 81, 125105 (2010))
- **Ladder approximation, static polarization**. J. Wang, H. A. Fertig and G. Murthy, PRL 104, 186401 (2010); O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 80, 165429 (2009)

  $\Rightarrow$ critical $\alpha_c \approx 1.62$
- **Gap equation, dynamical screening**. O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 81, 075429 (2010) $\Rightarrow$ critical $\alpha_c \approx 0.92$
- **Effect of Fermi velocity renormalization**. D. V. Khveshchenko, JPCM 21, 075303 (2009); J. Sabio, F. Sols and F. Guinea, PRB 82, 121413(R) (2010)
- **Ladder approximation, dynamical screening + electron self-energy corrections**.

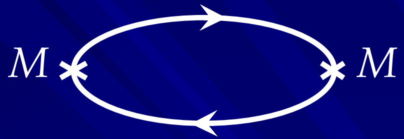
  J. G., PRB 85, 085420 (2012) $\Rightarrow$ critical $\alpha_c \approx 1.75$

But, what about the dynamical generation of a parity-breaking mass in graphene?

DYNAMICAL MASS GENERATION. RENORMALIZATION GROUP

The dynamical mass generation can be characterized by the divergence of the susceptibility

$$\Pi^{(m)}(\mathbf{r}) = \left\langle \Psi^{+}(\mathbf{r})\, M\, \Psi(\mathbf{r})\;\; \Psi^{+}(\mathbf{0})\, M\, \Psi(\mathbf{0}) \right\rangle \quad \sim$$



A divergence of $\Pi^{(m)}$ at $\mathbf{q} \to 0$ will imply the spontaneous development of a condensate

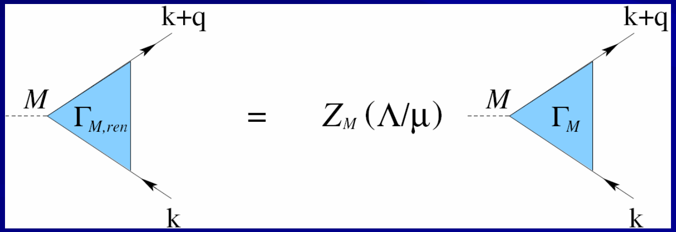
$$\left\langle \Psi^{+}(\mathbf{r})\, M\, \Psi(\mathbf{r}) \right\rangle \neq 0$$

The low-energy behavior of the susceptibility is:

$$\text{non-interacting theory} \qquad \Pi^{(m)}(\mathbf{q}, \omega = 0) \sim |\mathbf{q}|$$

$$\text{interacting theory} \qquad \Pi^{(m)}(\mathbf{q}, \omega = 0) \sim |\mathbf{q}| \left(\frac{\mu}{|\mathbf{q}|} \right)^{2\gamma}$$

The scaling of $\Pi^{(m)}$ is modified because the interactions introduce dependence on the cutoff Λ

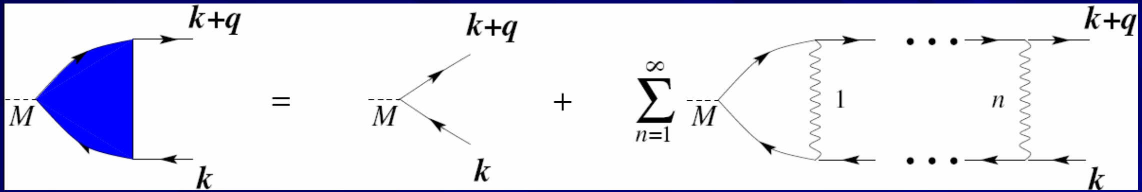


The independence of $\Pi^{(m)}(\mathbf{q},0) = (Z_M)^{-2}\, \Pi^{(m)}{}_{\text{ren}}(\mathbf{q},0)$ on μ leads to the equation

$$\left(\mu \frac{\partial}{\partial \mu} - 2\gamma \right) \Pi^{(m)}_{\text{ren}}(\mathbf{q},0) = 0 \qquad \Rightarrow \qquad \Pi^{(m)}_{\text{ren}}(\mathbf{q},0) \sim \mu^{2\gamma} \qquad\qquad \gamma = -\frac{\Lambda}{Z_M} \frac{\partial Z_M}{\partial \Lambda}$$

DYNAMICAL MASS GENERATION. LADDER APPROXIMATION

We can study the divergences of the vertex in the ladder approximation



Screening the interaction in the static RPA, we get the self-consistent equation

$$\Gamma_M(\mathbf{q}=\mathbf{0};\mathbf{k}) = M + ie^2\mu^\varepsilon \int \frac{d\omega_p}{2\pi}\frac{d^{2-\varepsilon}p}{(2\pi)^2}\,\gamma_0 G(\mathbf{p},\omega_p)\,\Gamma_M(\mathbf{q}=\mathbf{0};\mathbf{p})\,G(\mathbf{p},\omega_p)\gamma_0\,\frac{1}{2\kappa|\mathbf{p}-\mathbf{k}|} \qquad \kappa = 1+\frac{e^2}{8v_F}$$

A suitable way of extracting the scale dependence of Γ_M is to compute the integrals at dimension $D = 2 - \varepsilon$. The only dependence on μ comes from the need to introduce a dimensionful $e^2{}_0 = \mu^\varepsilon e^2$. The cutoff divergences in Γ_M now appear as poles $1/\varepsilon$, $1/\varepsilon^2$, …

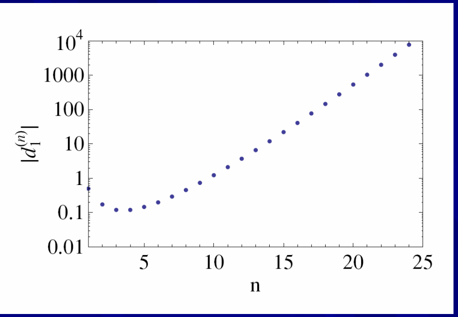
$$\Gamma_M(\mathbf{q};\mathbf{k})\Big|_{\mathrm{ren}} = Z_M\,\Gamma_M(\mathbf{q};\mathbf{k}) \qquad Z_M = 1+\sum_{n=1}^{\infty}\frac{d_n(\lambda)}{\varepsilon^n} \qquad \lambda = \frac{e^2}{4\pi\kappa v_F}$$

The anomalous dimension is now

$$\gamma = \frac{\mu}{Z_M}\frac{\partial Z_M}{\partial\mu} = \frac{\mu}{Z_M}\frac{\partial\lambda}{\partial\mu}\frac{\partial Z_M}{\partial\lambda} = -\lambda\, d_1'(\lambda)$$

DYNAMICAL MASS GENERATION. LADDER APPROXIMATION

The anomalous dimension $\gamma = -\lambda\, d'_1(\lambda)$ can be computed perturbatively from the expansion



$$d_1(\lambda) = \sum_{n=1}^{\infty} d_1^{(n)} \lambda^n$$

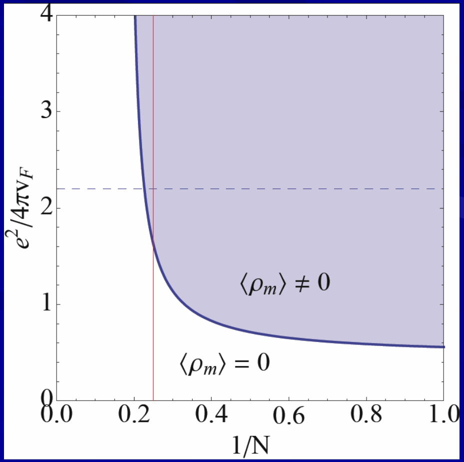
It turns out that the series for γ has a finite radius of convergence

$$\lambda_c \approx 0.456947$$ (J. G., JHEP 08, 27 (2012))

which coincides precisely with the critical coupling obtained from the gap equation

$$\lambda_c = 8\pi^2/(\Gamma(1/4))^4$$ (O. V. Gamayun *et al.*, PRB 80, 165429 (2009))

λ_c is the critical coupling for the dressed Coulomb potential, and its relation to the nominal coupling $\alpha = e^2/4\pi v_F$ depends on screening effects.

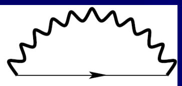


Under static RPA $$\lambda_c = \frac{\alpha_c}{1 + \frac{N\pi}{8}\alpha_c}$$

DYNAMICAL MASS GENERATION. LADDER APPROXIMATION

The above approach can be improved adding the effect of electron self-energy corrections

$$\tilde{v}_F(\mathbf{p}) = Z_v v_F + $$

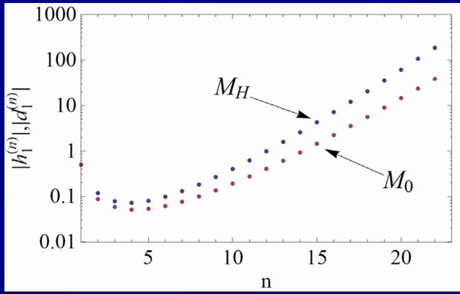


$$\Gamma_M(\mathbf{q}=\mathbf{0};\mathbf{k}) = M + ie^2\mu^\varepsilon \int \frac{d\omega_p}{2\pi}\frac{d^{2-\varepsilon}p}{(2\pi)^2}\gamma_0 G(\mathbf{p},\omega_p)\,\Gamma_M(\mathbf{q}=\mathbf{0};\mathbf{p})\,G(\mathbf{p},\omega_p)\gamma_0 \frac{1}{2\kappa|\mathbf{p}-\mathbf{k}|}$$

In this case it makes a difference to choose

$M_0 = \gamma_0$ or $M_H = i\,\gamma_1\,\gamma_2$

It turns out that the perturbative series diverges faster for M_H

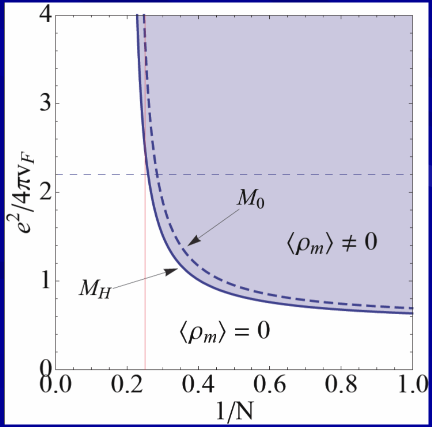


We obtain now the critical couplings

$\lambda_c \approx 0.5448$ for $M_0 = \gamma_0$

$\lambda_c \approx 0.508$ for $M_H = i\,\gamma_1\,\gamma_2$

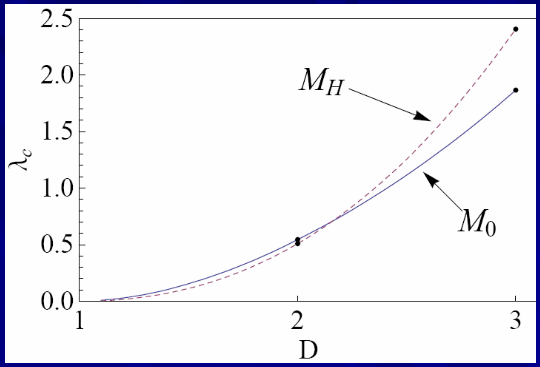
which are substantially larger than in the absence of electron self-energy corrections.



(J. G., arXiv:1211.3905)

DYNAMICAL MASS GENERATION AT GENERAL D

One can also renormalize the many-body theory at any spatial dimension D. Then, chiral symmetry breaking turns out to be the dominant instability at $D = 3$, but there is a crossover in favor of parity breaking at $D = 2$.



The fact that the dynamical breakdown of parity and chiral symmetry breaking do not take place at the same λ_c at $D = 2$ can be considered as an anomaly (a consequence of the regularization at $D = 2 - \varepsilon$).

Dimensional regularization is singled out as a method preserving the gauge invariance of the Dirac field theory. But it is worthwhile to look at a different gauge invariant alternative given by the lattice regularization of the Dirac fermions

$$S = \int dt d^2 r\, \overline{\Psi}(\mathbf{r})\, i\left(\gamma_0 \partial_t - v_F \boldsymbol{\gamma} \cdot \nabla\right) \Psi(\mathbf{r})$$

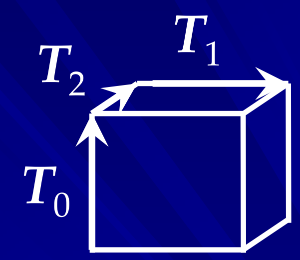
$$\Rightarrow \quad \sum_{\mathbf{n},\mu=0,1,2} \eta_\mu(\mathbf{n}) \left(\overline{\psi}(\mathbf{n})\, U(\mathbf{n}+\mathbf{e}_\mu)\, \psi(\mathbf{n}+\mathbf{e}_\mu) - \overline{\psi}(\mathbf{n})\, U^+(\mathbf{n})\, \psi(\mathbf{n}-\mathbf{e}_\mu)\right)$$

DYNAMICAL MASS GENERATION ON THE LATTICE

In the formulation of staggered fermions, the Dirac fermion components are spread over each unit cell of the lattice, and the mass matrices M_0 and M_H are realized in different ways:

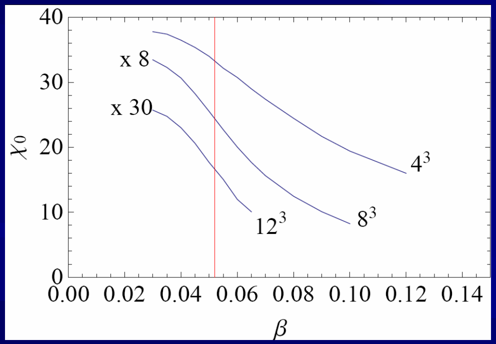
$$M_0 = \gamma_0 \quad \Rightarrow \quad S_0 = m_0\, \overline{\psi}(\mathbf{n})\psi(\mathbf{n})$$

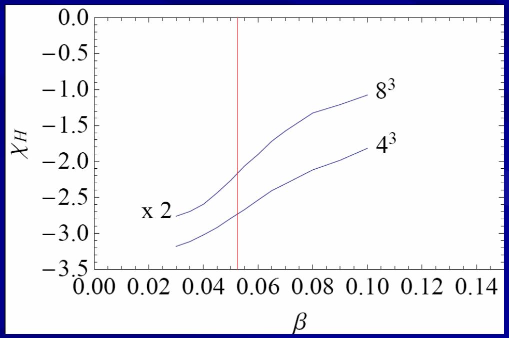
$$M_H = i\,\gamma_1\gamma_2 \quad \Rightarrow \quad S_H = m_H\; i\overline{\psi}(\mathbf{n})\sum_{ijk}\eta_0\eta_1\eta_2 T_i T_j T_k\; \psi(\mathbf{n})$$



One can compute for instance the susceptibilities

$$\chi_0 = \left.\frac{\partial\left\langle \overline{\psi}(\mathbf{n})\psi(\mathbf{n})\right\rangle}{\partial m_0}\right|_{m_0=0} \qquad \chi_H = \left.\frac{\partial\left\langle i\overline{\psi}(\mathbf{n})\sum\eta_0\eta_1\eta_2 T_i T_j T_k\; \psi(\mathbf{n})\right\rangle}{\partial m_H}\right|_{m_H=0}$$





$$\beta \equiv \frac{v_F}{e^2}$$

χ_0 and χ_H remain regular at the transition (consistent with the solution of the gap equation) and show very close critical couplings ($\beta_c \lesssim 0.05$, $e^2/4\pi v_F \gtrsim 1.6$) for the two symmetry breakings.

In conclusion:

We have seen that the dynamical breakdown of parity (and time-reversal invariance) may have at least equal strength in graphene as the chiral symmetry breaking.

- in the ladder approximation, electron self-energy corrections increase significantly the critical coupling up to $\alpha_c \approx 2.51$ (much lower anyhow than that for chiral symmetry breaking, $\alpha_c \approx 3.78$)
- a more comprehensive consideration of different screening effects is achieved with the lattice gauge theory approach, leading to $\alpha_c \gtrsim 1.6$ (which compares well with the result $\alpha_c \approx 1.75$ from dynamical screening in the RPA)

It is still puzzling that the most sensible values found for α_c would imply the development of a gap in the spectrum of free-standing graphene samples, while there seems to be no experimental observation in that direction

$\Rightarrow$ possible development of more exotic effects from the nucleation of loop currents, domains with alternating orientation of the order parameter, etc.