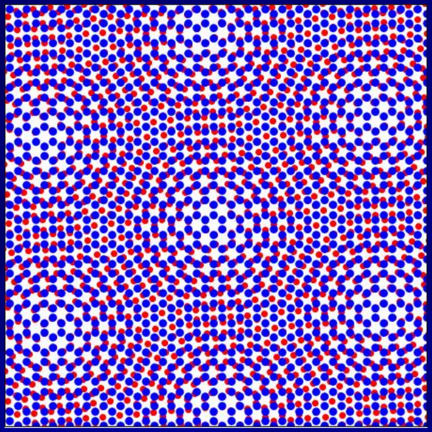
NON-ABELIAN GAUGE POTENTIALS IN GRAPHENE BILAYERS



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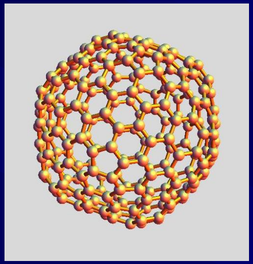
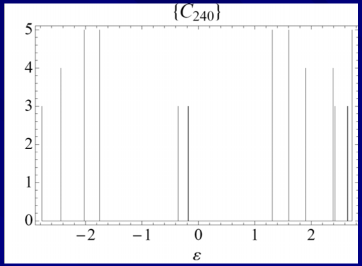
GAUGE POTENTIALS IN GRAPHENE

- Gauge potentials from topological defects (pentagonal, heptagonal rings)

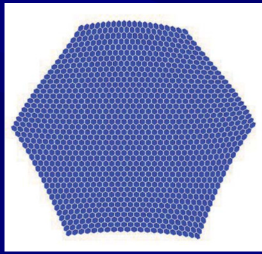
  (J. G., F. Guinea and M. A. H. Vozmediano, Nucl. Phys. B 406, 771 (1993))

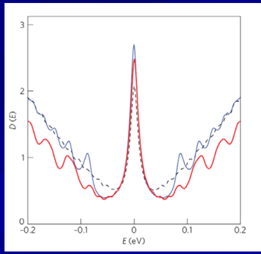




- Gauge potentials from smooth lattice deformations

  (F. Guinea, M. I. Katsnelson and A. K. Geim, Nature Phys. 6, 30 (2009))





- Gauge potentials from lattice mismatch in graphene bilayers

  (M. Mucha-Kruczynski *et al.*, Phys. Rev. B 84, 041404 (2011); Y.-W. Son *et al.*, Phys. Rev. B 84, 155410 (2011); E. Mariani *et al.*, arXiv:1110.2769) (see also J. Sun *et al.* Phys. Rev. Lett. 105, 156801 (2010))

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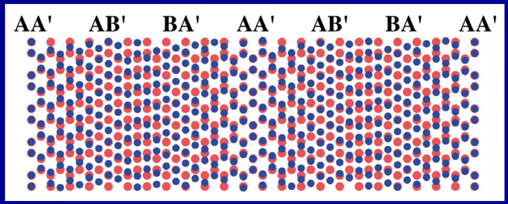
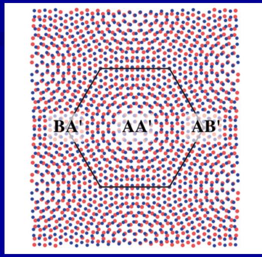
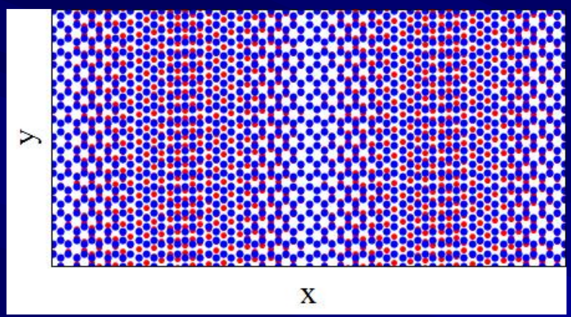
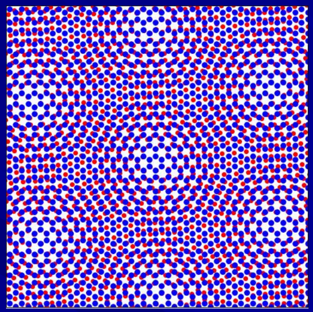


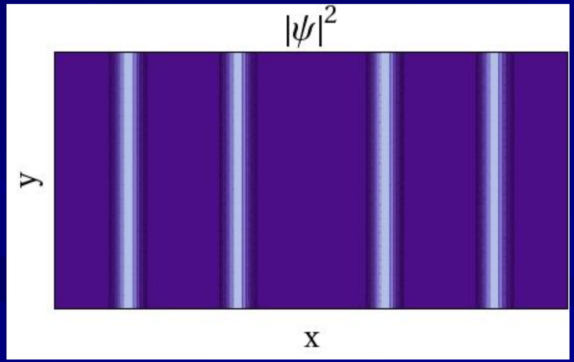


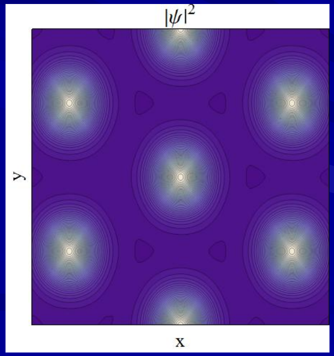
CONFINEMENT FROM NON-ABELIAN GAUGE POTENTIALS

It can be shown that graphene bilayers with distinctive Moiré patterns develop confined electronic states according to the structure of the superlattice





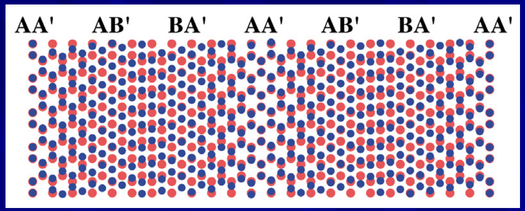




NON-ABELIAN GAUGE FIELDS IN GRAPHENE BILAYERS

We devise a continuum model for the low-energy electronic states in graphene bilayers

$$H = v_F \begin{pmatrix} 0 & -i(\partial_x - i\partial_y) & V_{AA'}(\mathbf{r}) & V_{AB'}(\mathbf{r}) \\ -i(\partial_x + i\partial_y) & 0 & V_{BA'}(\mathbf{r}) & V_{BB'}(\mathbf{r}) \\ V_{AA'}(\mathbf{r}) & V_{BA'}(\mathbf{r}) & 0 & -i(\partial_x - i\partial_y) \\ V_{AB'}(\mathbf{r}) & V_{BB'}(\mathbf{r}) & -i(\partial_x + i\partial_y) & 0 \end{pmatrix}$$



We can decompose the interlayer tunneling amplitudes in the form

$$V_{AB'}(\mathbf{r}) = -A_x(\mathbf{r}) + A_y(\mathbf{r}) \qquad V_{BA'}(\mathbf{r}) = -A_x(\mathbf{r}) - A_y(\mathbf{r})$$

so that A_x and A_y induce an off-diagonal shift of the momenta. We can write in compact form

$$H = v_F \boldsymbol{\sigma} \cdot (-i\partial - \hat{\mathbf{A}}) \ + \ v_F V_{AA'} \tau_1 \qquad \hat{\mathbf{A}} = (A_x \tau_1, A_y \tau_2)$$

The matrix-valued $\hat{\mathbf{A}}$ acts as a genuine gauge potential, giving rise to a Zeeman term

$$v_F^2 \left(-\partial^2 + i\partial \cdot \hat{\mathbf{A}} + 2i\hat{\mathbf{A}} \cdot \partial + A_x^2 + A_y^2 - \sigma_z \hat{F}_{xy} \right) \Psi = \varepsilon^2 \Psi$$

where the pseudospin is coupled to the non-Abelian field strength

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{\mathrm{A}}_\nu - \partial_\nu \hat{\mathrm{A}}_\mu - i\left[\hat{\mathrm{A}}_\mu , \hat{\mathrm{A}}_\nu\right]$$

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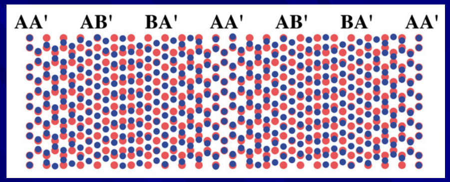
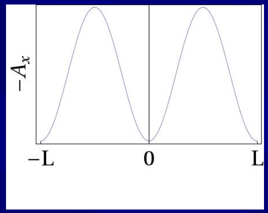
NON-ABELIAN GAUGE FIELDS IN SHEARED BILAYERS

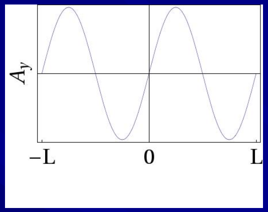
$$H = v_F \begin{pmatrix} 0 & -i(\partial_x - i\partial_y) & V_{AA'}(\mathbf{r}) & V_{AB'}(\mathbf{r}) \\ -i(\partial_x + i\partial_y) & 0 & V_{BA'}(\mathbf{r}) & V_{BB'}(\mathbf{r}) \\ V_{AA'}(\mathbf{r}) & V_{BA'}(\mathbf{r}) & 0 & -i(\partial_x - i\partial_y) \\ V_{AB'}(\mathbf{r}) & V_{BB'}(\mathbf{r}) & -i(\partial_x + i\partial_y) & 0 \end{pmatrix}$$



In the case of strained bilayers, the gauge potentials have the periodicity of the Moiré pattern

$$V_{AB'}(x) = -A_x(x) + A_y(x)$$
$$V_{BA'}(x) = -A_x(x) - A_y(x)$$

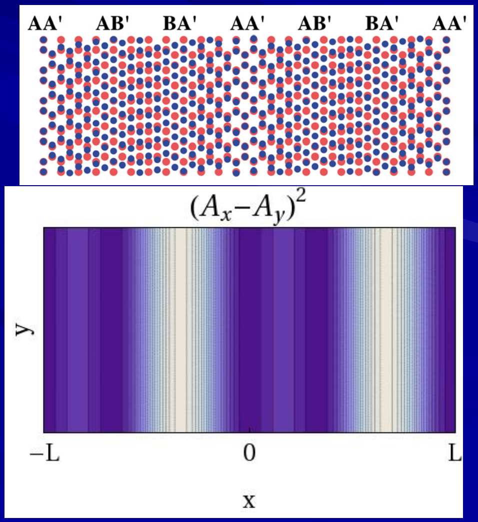




At large period *L*, there is an effective potential that has zeros at the center of *AA′* stacking, and either *AB′* or *BA′* stacking depending on the value of the pseudospin σ_z

$$v_F^2(-\partial^2 + i\partial\cdot\hat{\mathbf{A}} + 2i\hat{\mathbf{A}}\cdot\partial + \underbrace{A_x^2 + A_y^2}$$

$$-\sigma_z(\partial_x A_y \tau_2 - \underbrace{2A_x A_y \tau_3}))\,\Psi = \varepsilon^2\,\Psi$$



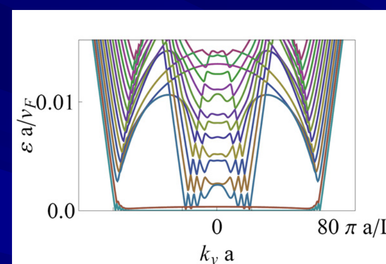
NON-ABELIAN GAUGE FIELDS IN SHEARED BILAYERS

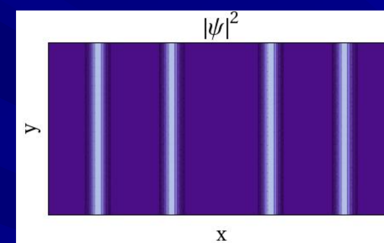
The precise pattern of confinement is found by diagonalizing the full hamiltonian:

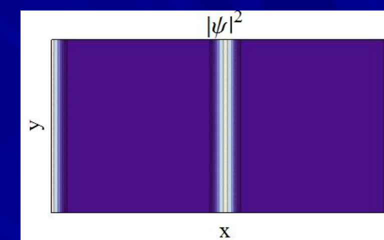
$$v_F\left[\sigma_x(-i\partial_x - A_x(x)\,\tau_1)+\sigma_y(k_y - A_y(x)\,\tau_2)+V_{AA'}(x)\,\tau_1\right]\Psi(x)=\varepsilon\,\Psi(x)$$

- For transverse momentum $k_y \neq 0$, the band structure is strongly reminiscent of that found in thick carbon nanotubes in a real perpendicular magnetic field

$k_y \neq 0$ ⟶

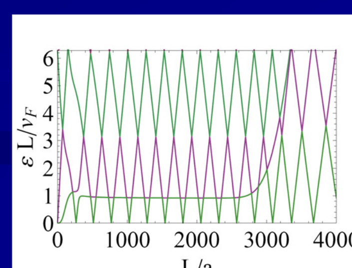






- At transverse momentum $k_y = 0$, we find that the lowest energy subband touches recurrently the level of zero energy, which is a genuine effect of the non-Abelian gauge potential

$k_y = 0$ ⟶



$$\Psi(x)_{\varepsilon=0} = P e^{i\int_0^x dx'\left[A_x(x')\tau_1 - iA_y(x')\tau_2\right]}\,\Psi(0)$$

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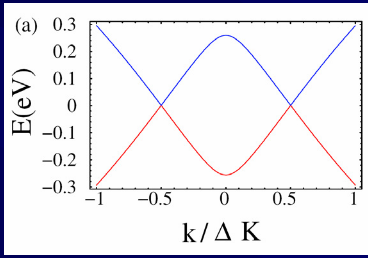
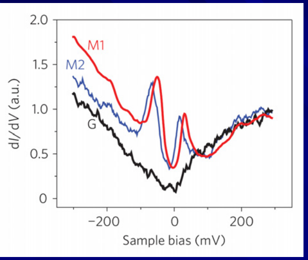
ELECTRONIC PROPERTIES OF TWISTED BILAYERS

- Low-energy theory in the continuum limit

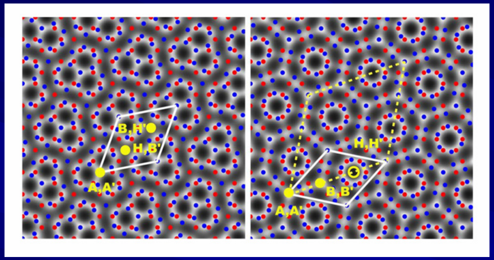


J. M. B. Lopes dos Santos, N. M. R. Peres and A. H. Castro Neto, Phys. Rev. Lett. 99, 256802 (2007)



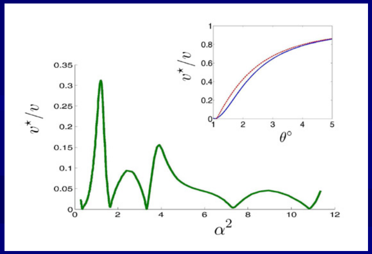
G. Li *et al.*, Nat. Phys. 6, 109 (2010)

- Classification of twisted bilayers

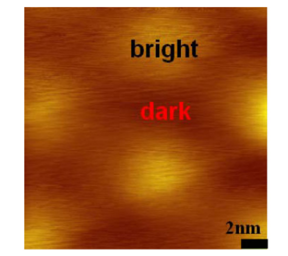


E. J. Mele, Phys. Rev. B 81, 161405(R) (2010)

- Fermi velocity renormalization at small twist angles



R. Bistritzer and A. H. MacDonald, Proc. Natl. Acad. Sci. 108, 12233 (2011)

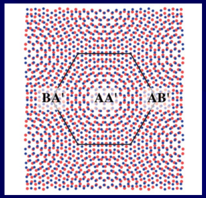


A. Luican *et al.*, Phys. Rev. Lett. 106, 126802 (2011)

R. de Gail *et al.*, Phys. Rev. B 84, 045436 (2011); M. Kindermann and E. J. Mele, Phys. Rev. B 84, 161406(R) (2011); E. Suárez Morell *et al.*, Phys. Rev. B 84, 195421 (2011); E. J. Mele, Phys. Rev. B 84, 235439 (2011); J. M. B. Lopes dos Santos *et al.*, arXiv:1202.1088 .

NON-ABELIAN GAUGE FIELDS IN TWISTED BILAYERS

In the continuum limit (small rotation angles), the low-energy states of twisted bilayers are obtained from the hamiltonian



$$H = v_F \begin{pmatrix} 0 & -i(\partial_x - i\partial_y) + i\Delta K/2 & V_{AA'}(\mathbf{r}) & V_{AB'}(\mathbf{r}) \\ -i(\partial_x + i\partial_y) - i\Delta K/2 & 0 & V_{BA'}(\mathbf{r}) & V_{BB'}(\mathbf{r}) \\ V^*_{AA'}(\mathbf{r}) & V^*_{BA'}(\mathbf{r}) & 0 & -i(\partial_x - i\partial_y) - i\Delta K/2 \\ V^*_{AB'}(\mathbf{r}) & V^*_{BB'}(\mathbf{r}) & -i(\partial_x + i\partial_y) + i\Delta K/2 & 0 \end{pmatrix}$$

We define now the gauge fields by

$$V_{AB'} = -A_{1x} + A_{2y} \quad + \quad i(A_{2x} + A_{1y})$$
$$V_{BA'} = -A_{1x} - A_{2y} \quad + \quad i(A_{2x} - A_{1y})$$

Introducing the generators $\{\tau_i\}$ of the SU(2) gauge group, we can write in compact form

$$H = v_F \boldsymbol{\sigma} \cdot (-i\partial - \tau_3 \Delta\mathbf{K}/2 - \hat{\mathbf{A}}) \quad + \quad v_F \hat{\Phi} \qquad \hat{\mathbf{A}} = (A_{1x}\tau_1 + A_{2x}\tau_2, A_{1y}\tau_1 + A_{2y}\tau_2)$$
$$\hat{\Phi} = \mathrm{Re}(V_{AA'})\,\tau_1 - \mathrm{Im}(V_{AA'})\,\tau_2$$

We can remove the $\Delta\mathbf{K}$ mismatch of the Dirac cones by means of a gauge transformation

$$\Psi = e^{\frac{i}{2}\tau_3 \Delta\mathbf{K}\cdot\mathbf{r}}\,\tilde{\Psi} \qquad \Rightarrow \qquad H = v_F \boldsymbol{\sigma} \cdot (-i\partial - \hat{\mathbf{A}}) \quad + \quad v_F \hat{\Phi}$$

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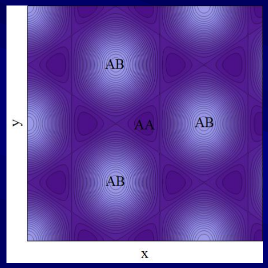
NON-ABELIAN GAUGE FIELDS IN TWISTED BILAYERS

An idea of the pattern of confinement in the twisted bilayer can be obtained (at $\Phi = 0$) from

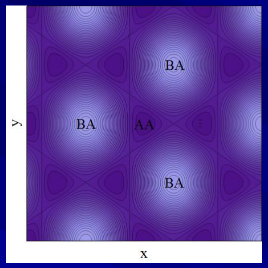
$$v_F^2 \left(-\partial^2 + i\partial \cdot \hat{\mathbf{A}} + 2i\hat{\mathbf{A}} \cdot \partial + \underbrace{A_{1x}^2 + A_{2x}^2 + A_{1y}^2 + A_{2y}^2} \right.$$

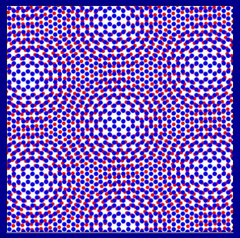
$$\left. -\sigma_z (\partial_x A_{1y} \tau_1 + \partial_x A_{2y} \tau_2 - \partial_y A_{1x} \tau_1 - \partial_y A_{2x} \tau_2 + \underbrace{2 A_{1x} A_{2y} \tau_3 - 2 A_{2x} A_{1y} \tau_3}) \right) \Psi = \varepsilon^2 \Psi$$

The combination $(A_{1x} \pm A_{2y})^2 + (A_{2x} \mp A_{1y})^2$ acts as an effective potential, that keeps the charge density away from the regions of stacking *AB′* or *BA′*

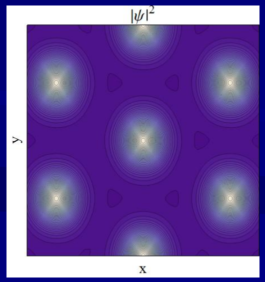


$$(A_{1x} \pm A_{2y})^2 + (A_{2x} \mp A_{1y})^2$$

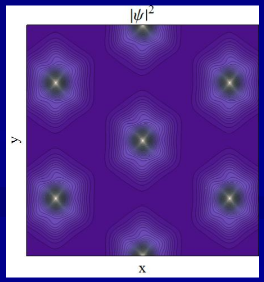




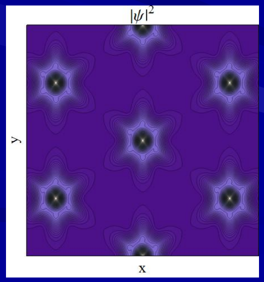
This trend is reinforced by the field strength of $\hat{\mathbf{A}}$, leading to localization around *AA′* stacking



$\theta \approx 1.0°$



$\theta \approx 0.5°$

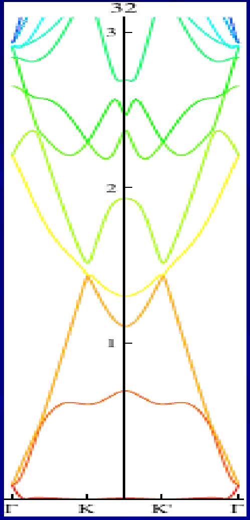


$\theta \approx 0.3°$

(see also G. T. de Laissardière *et al.*, Nano Letters 10, 804 (2010))

NON-ABELIAN GAUGE FIELDS IN TWISTED BILAYERS

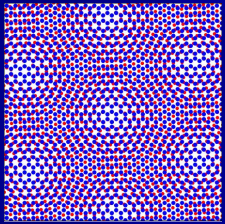
Why the magic angles?
The first (largest) angle at which the lowest subband becomes flat corresponds to the situation in which the pseudomagnetic length l_B starts to fit in the unit cell of the superlattice



$$l_B \approx \sqrt{\frac{v_F L}{w}} \sim L$$

Actually, the lowest subband becomes flat at the same twist angle that the integral of the field strength over the unit cell equals the quantum of flux ($2\pi\hbar$), up to rotations in SU(2) space:

$$\hat{\phi} = \int_{\text{unit cell}} d^2 r\, \hat{F}_{xy} \approx 2\pi\hbar \begin{pmatrix} 0 & -i\, e^{i2\pi n/3} \\ i\, e^{-i2\pi n/3} & 0 \end{pmatrix}$$



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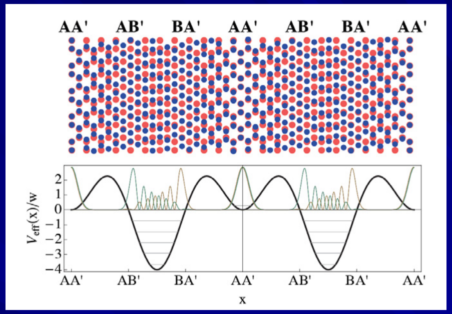
Smaller magic angles are not so simple to characterize, as the scalar potential Φ from the AA' regions starts to have then significant influence on the low-energy subbands.

NON-ABELIAN GAUGE FIELDS IN GRAPHENE BILAYERS

In conclusion,

- We have seen that the mismatch in the registry of graphene bilayers leads to patterns of electron localization that can be understood in terms of confinement by a non-Abelian gauge potential



- The characteristic feature of the non-Abelian gauge field is that it leads to the recurrent development of quite flat bands near zero energy, due to an effect of interference between different SU(2) components