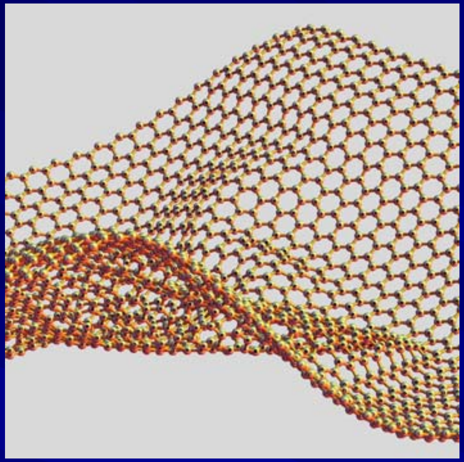
# EXCITONIC GAP FROM LONG-RANGE COULOMB INTERACTION IN GRAPHENE



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## Is it possible to have a gap generated spontaneously by interactions in graphene?

There have been already several analyses using different approximations:

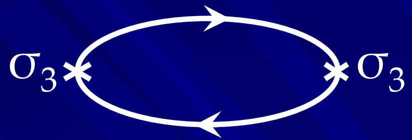
- **Gap equation, 1/$N$ approximation**. D. V. Khveshchenko, PRL 87, 246802 (2001); E. V. Gorbar, V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, PRB 66, 045108 (2002).
- **Renormalization group, 1/$N$ approximation**. J. E. Drut and D. T. Son, PRB 77, 075115 (2008); J. González, PRB 82, 155404 (2010) $\Rightarrow$ critical $N_c = 32/\pi^2$
- **Lattice field theory**. J. E. Drut and T. A. Lähde, PRL 102, 026802 (2009); PRB 79, 241405(R) (2009) $\Rightarrow$ critical $\alpha_c \approx 1.08$

  (see also W. Armour, S. Hands, C. Strouthos, PRB 81, 125105 (2010))
- **Ladder approximation, static polarization**. J. Wang, H. A. Fertig and G. Murthy, PRL 104, 186401 (2010); O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 80, 165429 (2009) $\Rightarrow$ critical $\alpha_c \approx 1.62$
- **Gap equation, dynamical screening**. O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 81, 075429 (2010) $\Rightarrow$ critical $\alpha_c \approx 0.92$
- **Ladder approximation, dynamical screening**. J. G., arXiv:1103.3650 $\Rightarrow$ critical $\alpha_c \approx 0.99$
- **Effect of Fermi velocity renormalization**. D. V. Khveshchenko, JPCM 21, 075303 (2009); J. Sabio, F. Sols and F. Guinea, PRB 82, 121413(R) (2010).
- **Gross-Neveu interactions**. I. F. Herbut, V. Juricic and O. Vafek, PRB 80, 075432 (2009); V. Juricic, I. F. Herbut and G. W. Semenoff, PRB 80, 081405 (2009).

# EXCITONIC INSTABILITY. RENORMALIZATION GROUP

The excitonic instability can be characterized by the divergence of the susceptibility

$$\Pi^{(3)}(\mathbf{r}) = \left\langle \Psi^{+}(\mathbf{r})\,\sigma_3\Psi(\mathbf{r})\;\; \Psi^{+}(\mathbf{0})\,\sigma_3\Psi(\mathbf{0}) \right\rangle \quad \sim$$




A divergence of $\Pi^{(3)}$ at $\mathbf{q} \to 0$ will imply the spontaneous development of a condensate

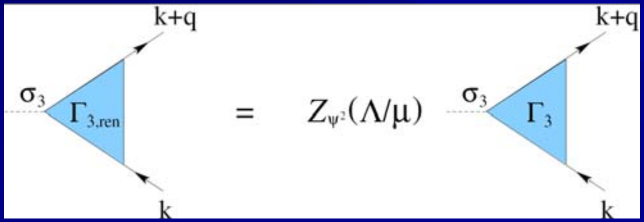
$$\left\langle \Psi^{+}(\mathbf{r})\,\sigma_3\Psi(\mathbf{r}) \right\rangle \neq 0$$

The low-energy behavior of the susceptibility is:

non - interacting theory $\quad \Pi^{(3)}(\mathbf{q}, \omega = 0) \sim |\mathbf{q}|$

interacting theory $\quad \Pi^{(3)}(\mathbf{q}, \omega = 0) \sim |\mathbf{q}| \left( \dfrac{|\mathbf{q}|}{\mu} \right)^{-2\gamma}$

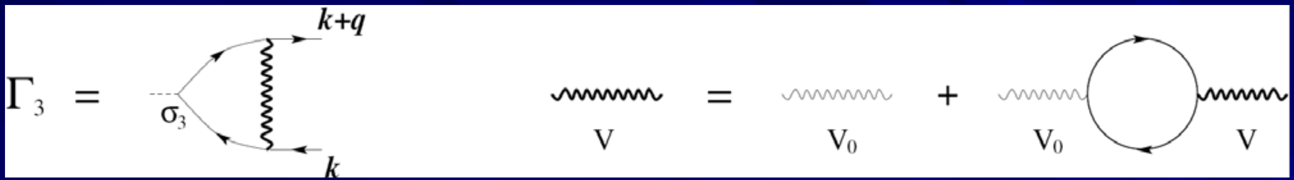
The scaling of $\Pi^{(3)}$ is modified because the interactions introduce dependence on the cutoff $\Lambda$




The independence of $\Pi^{(3)}(\mathbf{q},0) = (Z_{\psi^2})^{-2}\, \Pi^{(3)}_{\mathrm{ren}}(\mathbf{q},0)$ on $\mu$ leads to the equation

$$\left( \mu \frac{\partial}{\partial \mu} - 2\gamma \right) \Pi^{(3)}_{\mathrm{ren}}(\mathbf{q},0) = 0 \qquad \Rightarrow \qquad \Pi^{(3)}_{\mathrm{ren}}(\mathbf{q},0) \sim \mu^{2\gamma} \qquad \gamma = \mu \frac{\partial \log Z_{\psi^2}}{\partial \mu} = -\Lambda \frac{\partial \log Z_{\psi^2}}{\partial \Lambda}$$

# EXCITONIC INSTABILITY. 1/*N* APPROXIMATION




The question is to compute the logarithmic dependence on the cutoff $\Lambda$ when the interaction is screened with the polarization $\chi(\mathbf{q},\omega) = (N/16)\mathbf{q}^2 / \sqrt{v_F^2\mathbf{q}^2 - \omega^2}$

$$\Gamma_3(\mathbf{q};\mathbf{k}) = \sigma_3 + i\int \frac{d^2p}{(2\pi)^2}\int \frac{d\omega_p}{2\pi} G_0(\mathbf{p},\omega_p)\sigma_3 G_0(\mathbf{p}+\mathbf{q},\omega_p)\frac{e^2}{2|\mathbf{p}-\mathbf{k}| + e^2\chi(\mathbf{p}-\mathbf{k},\omega_p-\omega_k)}$$

We get the cutoff dependence

$$\Gamma_3(\mathbf{q};\mathbf{k}) = \sigma_3 + \sigma_3\frac{8}{\pi^2 N} g \frac{\arccos(g)}{\sqrt{1-g^2}}\log(\Lambda) \qquad g \equiv \frac{N}{32}\frac{e^2}{v_F}$$

But part of the divergence comes from simple renormalization of the quasiparticle weight, since the vertex with the operator $\sigma_0$ is already logarithmically divergent

$$\Gamma_0(\mathbf{q};\mathbf{k}) = \sigma_0 + \sigma_0\frac{8}{\pi^2 N}\left(\frac{\pi}{g} - \frac{2-g^2}{g}\frac{\arccos(g)}{\sqrt{1-g^2}} - 2\right)\log(\Lambda)$$

(J. G., F. Guinea and M.A.H. Vozmediano, Phys. Rev. B 59, 2474 (1999))

# EXCITONIC INSTABILITY. 1/$N$ APPROXIMATION

The vertex can be made finite by taking

$$\Gamma_3(\mathbf{q};\mathbf{k})\big|_{\mathrm{ren}} = Z_{\psi^2} Z_{\psi} \Gamma_3(\mathbf{q};\mathbf{k}) \qquad Z_{\psi^2} = 1 - \frac{8}{\pi^2 N}\left(2 + \frac{2}{g}\frac{\arccos(g)}{\sqrt{1-g^2}} - \frac{\pi}{g}\right)\log(\Lambda/\mu)$$

In analogous fashion, the susceptibility

$$\Pi^{(3)}(\mathbf{q},\omega_q)\big|_{\mathrm{ren}} \sim Z_{\psi^2}^2 \Pi^{(3)}(\mathbf{q},\omega_q)$$

This means that the susceptibility gets an anomalous dependence

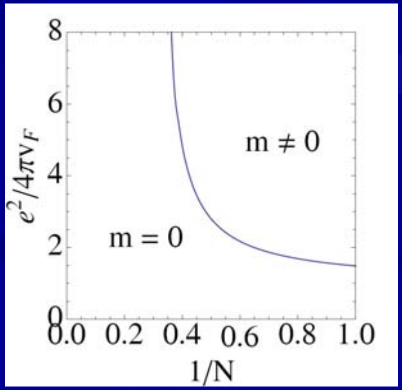
$$\Pi^{(3)}(\mathbf{q},\omega=0)\big|_{\mathrm{ren}} \sim |\mathbf{q}|\left(\frac{|\mathbf{q}|}{\mu}\right)^{-2\gamma} \qquad \gamma = \mu\frac{\partial \log Z_{\psi^2}}{\partial\mu} = \frac{8}{\pi^2 N}\left(2 + \frac{2}{g}\frac{\arccos(g)}{\sqrt{1-g^2}} - \frac{\pi}{g}\right)$$

The susceptibility blows up at $\mathbf{q}\to 0$ when

$$1 - 2\gamma < 0$$

This is the signature of the excitonic instability, which allows to identify the phase with

$$\langle \Psi^+(\mathbf{r})\,\sigma_3\Psi(\mathbf{r})\rangle \neq 0$$

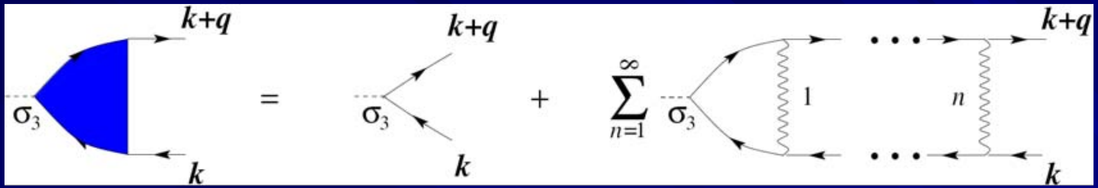



$$N_c = \frac{32}{\pi^2}$$

(J. G., PRB 82, 155404 (2010))

# EXCITONIC INSTABILITY. LADDER APPROXIMATION

We can go beyond the $1/N$ expansion with the ladder approximation




Screening the interaction in the static limit, we get the self-consistent equation

$$\Gamma_3(\mathbf{0};\mathbf{k}) = \sigma_3 + \pi\,\lambda \int \frac{d^2 p}{(2\pi)^2}\,\Gamma_3(\mathbf{0};\mathbf{p})\,\frac{1}{|\mathbf{p}|}\,\frac{1}{|\mathbf{p}-\mathbf{k}|} \qquad \lambda = \frac{\alpha}{1+\frac{N\pi}{8}\alpha} \qquad (\alpha = e^2/4\pi v_F)$$

A suitable way of extracting the scale dependence of $\Gamma_3$ is to compute the integrals at dimension $d = 2 - \varepsilon$. The only dependence on $\mu$ comes from the need to introduce a dimensionful $\lambda_0 = \mu^{\varepsilon}\lambda$. The ultraviolet divergences in $\Gamma_3$ now appear as poles $1/\varepsilon$, $1/\varepsilon^2$, …

$$\Gamma_3(\mathbf{q};\mathbf{k})\big|_{\mathrm{ren}} = Z_{\psi^2}\,\Gamma_3(\mathbf{q};\mathbf{k}) \qquad Z_{\psi^2} = 1 + \sum_{n=1}^{\infty} \frac{c_n(\lambda)}{\varepsilon^n}$$

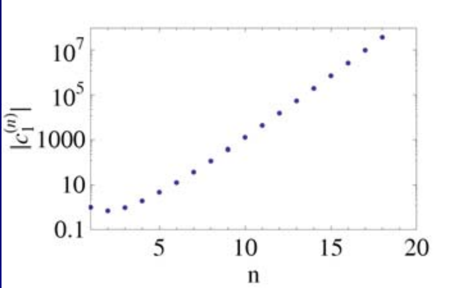
The anomalous dimension is now

$$\gamma = \mu \frac{\partial \log Z_{\psi^2}}{\partial \mu} = \mu \frac{\partial \lambda}{\partial \mu} \frac{\partial \log Z_{\psi^2}}{\partial \lambda} = -\,\lambda \frac{dc_1(\lambda)}{d\lambda}$$

# EXCITONIC INSTABILITY. LADDER APPROXIMATION.

The anomalous dimension $\gamma = -\lambda\, dc_1(\lambda)/d\lambda$ can be computed perturbatively from the expansion

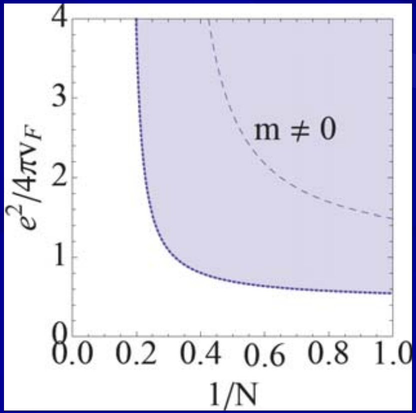
$$c_1(\lambda) = \sum_{n=1}^{\infty} c_1^{(n)} \lambda^n$$




It turns out that the series for $\gamma$ has a finite radius of convergence, $\lambda_c \approx 0.45$, at which the anomalous dimension diverges (J. G., PRB 82, 155404 (2010)).
The boundary of the excitonic instability corresponds now to

$$\lambda_c = \frac{\alpha_c}{1 + \frac{N\pi}{8}\alpha_c} \qquad (\alpha = e^2/4\pi v_F)$$

leading to a much more extended region with exciton condensation in the $(N,\alpha)$ phase diagram




$(\alpha_c \approx 1.53$ for $N = 4)$

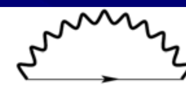
# EXCITONIC INSTABILITY. LADDER APPROXIMATION

The above approach can be improved adding the effect of electron self-energy corrections

$$\Gamma_3(\mathbf{0};\mathbf{k}) = \sigma_3 + \pi\,\lambda v_F \int \frac{d^2 p}{(2\pi)^2}\,\Gamma_3(\mathbf{0};\mathbf{p})\,\frac{1}{\tilde{v}_F(\mathbf{p})|\mathbf{p}|}\,\frac{1}{|\mathbf{p}-\mathbf{k}|}$$

$$\lambda = \frac{\alpha}{1+\frac{N\pi}{8}\alpha}$$

$$\tilde{v}_F(\mathbf{p}) = Z_v\, v_F \quad +$$



In this case the self-energy gets a pole in $\varepsilon = 2 - d$, that we have to subtract with $Z_v = 1 - b_1/\varepsilon$, leading to a renormalization of the coupling

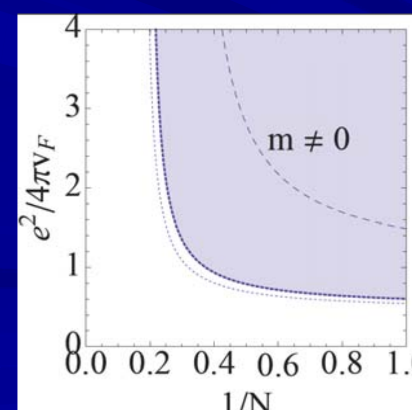
$$\lambda_0 = \mu^\varepsilon \frac{\lambda}{1-\frac{b_1(\lambda)}{\varepsilon}}$$

This changes the position of the poles in

$$Z_{\psi^2} = 1 + \sum_{n=1}^{\infty} \frac{c_n(\lambda)}{\varepsilon^n}$$

and the radius of convergence in $\gamma = -\lambda\, dc_1(\lambda)/d\lambda$
is shifted to a larger value $\lambda_c \approx 0.49$

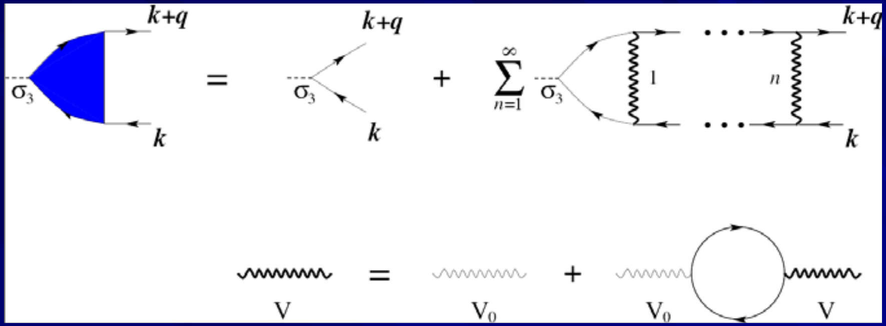
($\alpha_c \approx 2.09$ for $N = 4$)




(J. G., arXiv:1103.3650)

# EXCITONIC INSTABILITY. DYNAMICAL SCREENING

We can also study the excitonic instability when the Coulomb interaction is screened with the dynamical polarization:




In this case, the ladder approximation for the vertex is encoded into the integral equation

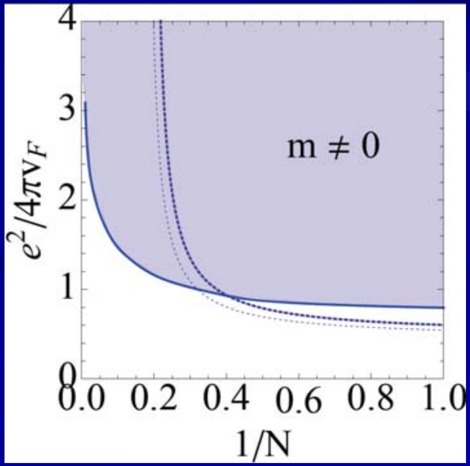
$$\Gamma_3(0;\mathbf{k},\omega_k) = \sigma_3 + \int \frac{d^2p}{(2\pi)^2} \int \frac{d\omega_p}{2\pi} \Gamma_3(0;\mathbf{p},\omega_p) \, \frac{1}{v_F^2\mathbf{p}^2 + \omega_p^2} \, \frac{e^2}{2|\mathbf{p}-\mathbf{k}| + e^2 \chi(\mathbf{p}-\mathbf{k}, i\omega_p - i\omega_k)}$$

One observes again that the vertex blows up at a critical coupling $\alpha_c$ for exciton condensation.

By following the dependence on the high-energy cutoff $\Lambda$, one can make use of the scaling law

$$\alpha_c(\Lambda) = \alpha_c(\infty) + \frac{c}{\Lambda^\nu}$$

$(\alpha_c(\infty) \approx 0.99 \;\; \text{for} \;\; N = 4)$




(J. G., arXiv:1103.3650)

In conclusion:

We have incorporated different many-body corrections to the vertex for exciton condensation in the theory of $N$ Dirac fermions with Coulomb interaction:

- in the ladder approximation, the static screening of the Coulomb interaction leads to $\alpha_c \approx 1.53$ for $N = 4$
- incorporating electron self-energy corrections in the ladder approximation weakens the exciton instability, leading to $\alpha_c \approx 2.09$ for $N = 4$
- supplementing the ladder approximation with the dynamical screening of the Coulomb interaction shows that the static limit overestimates the screening effects, and that a more reliable value is $\alpha_c \approx 0.99$ for $N = 4$

It is still puzzling that the values found for $\alpha_c$ would imply the development of a gap in the spectrum of free-standing graphene samples, while there seems to be no experimental observation in that direction

$\Rightarrow$ possible effect of Fermi velocity scaling at low energies, with the consequent reduction of the effective values of the coupling $\alpha = e^2/4\pi v_F$