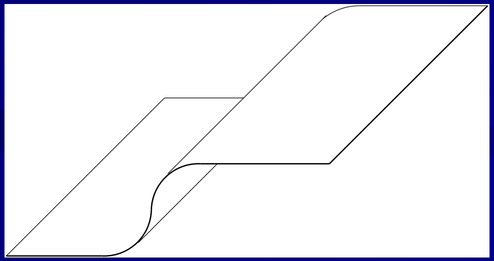
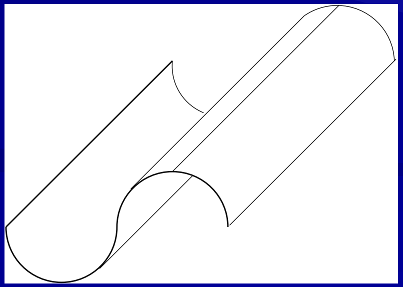
Quantum Hall effect in carbon nanotubes and curved graphene strips

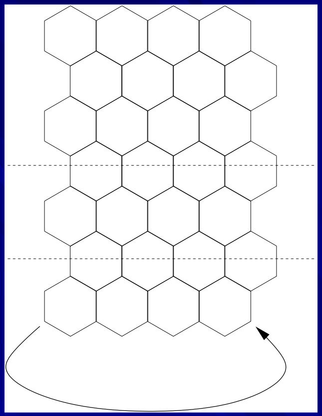
J. González and E. Perfetto

- carbon nanotubes have “edge” states in transverse magnetic fields
- the longitudinal currents from the edge states are quantized
- the Hall conductivity is quantized in even steps of $2e^2/h$
- more general “edge” states arise in curved graphene strips





CONTINUUM LIMIT OF 2D CARBON LATTICES AND NANOTUBES

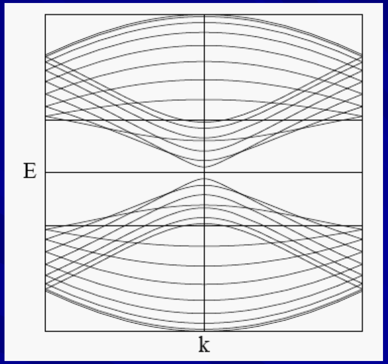


From the tight-binding hamiltonian

$$H_{tb} = -t\sum_{r,r'}\psi^{+}(\mathbf{r}')\exp(i(e/\hbar c)\int_{r}^{r'}\mathbf{A}\cdot d\mathbf{l})\psi(\mathbf{r})$$

the continuum limit introduces as many fields as atoms are present in the unit cell.

The hamiltonian in the space of subbands becomes:



$$H_{p,p'}\big|_{B=0} = \delta_{p,p'}\, t \begin{pmatrix} 0 & 1 & 0 & z_p^{*}e^{-i3ka} \\ 1 & 0 & z_p & 0 \\ 0 & z_p^{*} & 0 & 1 \\ z_p e^{i3ka} & 0 & 1 & 0 \end{pmatrix}$$

$$z_p = \cos(2\pi p/N)$$

or projecting onto the low-energy dispersive branches

$$H_{p,p'}\big|_{B=0} = \delta_{p,p'}\begin{pmatrix} v_F\hbar k & \Delta_p \\ \Delta_p & -v_F\hbar k \end{pmatrix}$$

$$\Delta_p = t\left|1-2\cos(\pi p/N)\right|$$

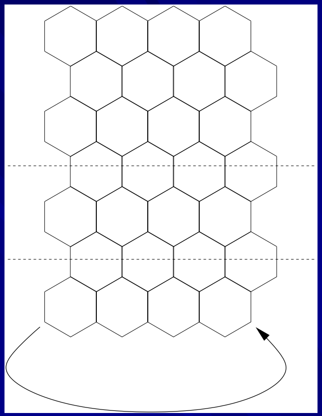
which is the hamiltonian for 1D massive Dirac fermions

EFFECTIVE FIELD THEORY OF CARBON NANOTUBE

The vector potential has a modulation around the nanotube

$$\mathbf{A} = (0, BR\sin(x/R))$$

and the continuum limit is obtained as $(e/\hbar c)BRa \ll 1$



$$\exp\left(i\frac{e}{\hbar c}\int_r^{r'} \mathbf{A}\cdot \mathbf{dl} \right) \approx 1 + i\eta \frac{e}{\hbar c} BRa\sin\left(\frac{x}{R}\right) + \dots$$

Projecting again onto the low-energy subband space:

$$H_{p,p'} = \delta_{p,p'} \begin{pmatrix} v_F \hbar k & \Delta_p \\ \Delta_p & -v_F\hbar k \end{pmatrix} + \delta_{p',p\pm 1} \begin{pmatrix} \pm i v_F (e/c)BR/2 & 0 \\ 0 & \mp i v_F (e/c) BR/2 \end{pmatrix}$$

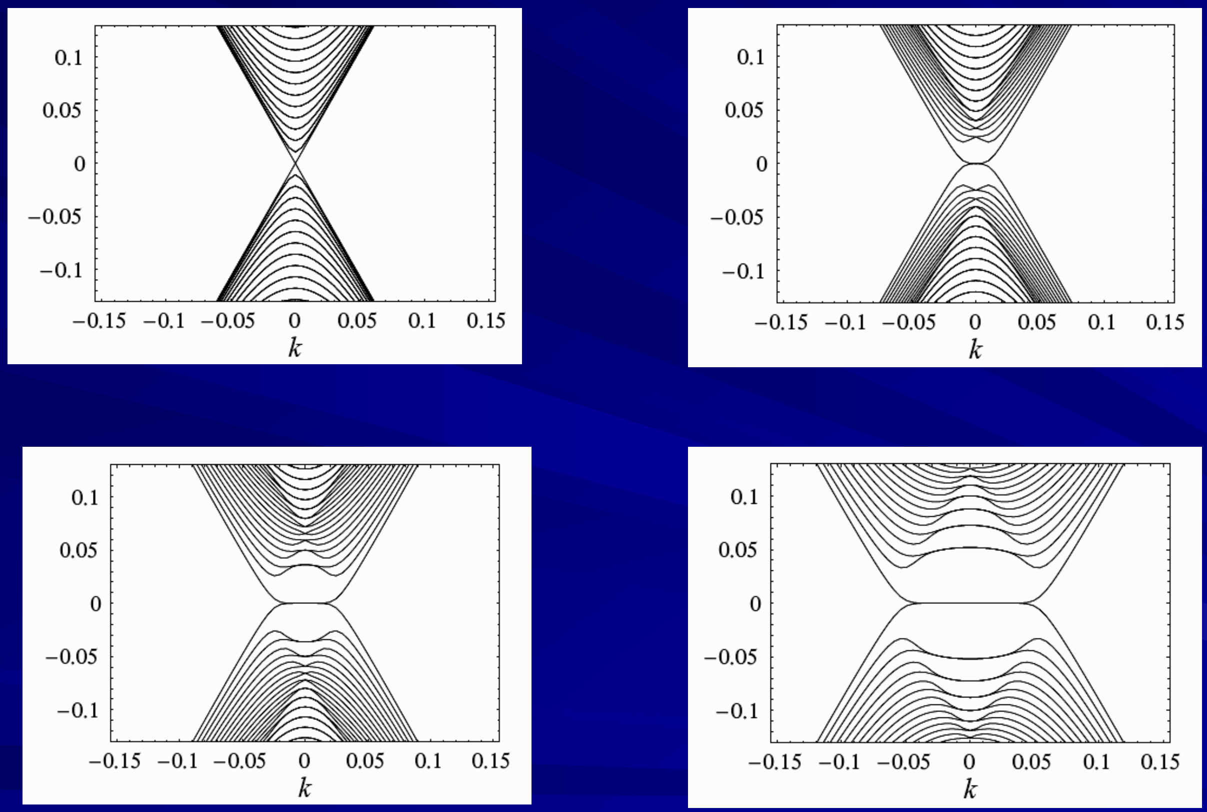
This combines nicely when transforming back to the real-space angular variable

$$H = \begin{pmatrix} v_F \hbar k + v_F (e/c) BR\sin(\theta) & i\frac{\hbar v_F}{a}\partial_\theta \\ i\frac{\hbar v_F}{a}\partial_\theta & -v_F\hbar k - v_F(e/c)BR\sin(\theta) \end{pmatrix}$$

The effect of the magnetic field enters through the usual gauge coupling to Dirac fermions (see H.-W. Lee and D. S. Novikov, Phys. Rev. B **68,** 155402 (2003)) but the coupling is not in general so simple in a curved geometry (E. Perfetto, J. González, F. Guinea, S. Bellucci and P. Onorato, Phys. Rev. B **76,** 125430 (2007)).

LANDAU SUBBANDS IN CARBON NANOTUBES

With the continuum field theory, one can study the development of Landau subbands in thick carbon nanotubes. For $R = 20$ nm and magnetic field strength $B = 0, 5, 10, 20$ T, we get the sequence of band structures (E. Perfetto *et al.*, Phys. Rev. B **76**, 125430 (2007))



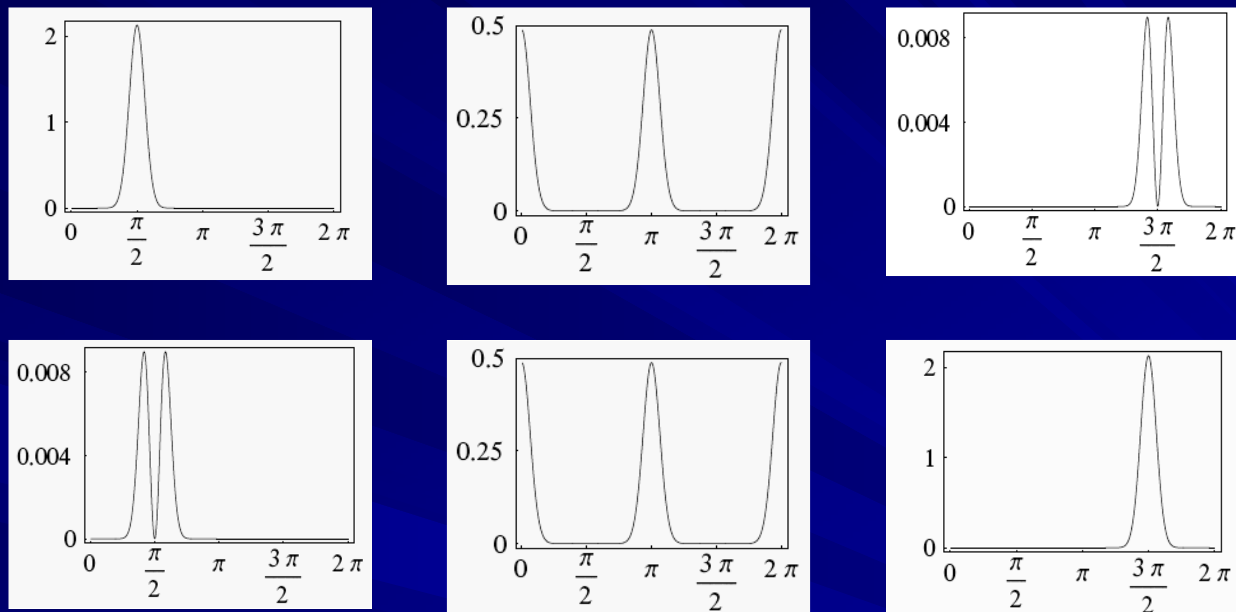
The magnetic field $B = 20$ T corresponds to $(e/\hbar c)BRa \approx 0.1$ (so that the continuum limit is justified) and magnetic length $\sqrt{\hbar c/eB} \approx 5.7\,\text{nm}$

EDGE STATES IN CARBON NANOTUBES

The important point is that the states in the dispersive branches of the different subbands are localized at the flanks of the nanotube

$|\psi_R|^2$

$|\psi_L|^2$



$k = 0.15$ $k = 0.0$ $k = -0.15$

The "edge" states carry currents in the longitudinal direction, whose measure is given by the continuity equation from the Dirac operator:

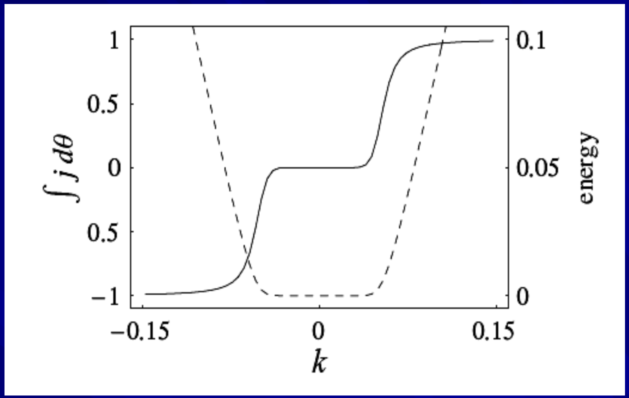
$$\partial_t(\psi_R^+\psi_R + \psi_L^+\psi_L) = v_F\,\partial_x(\psi_R^+\psi_R - \psi_L^+\psi_L)$$

CURRENT QUANTIZATION IN CARBON NANOTUBES

The longitudinal current is given by $j = v_F\,(\psi_R^+\psi_R - \psi_L^+\psi_L)$

It can be shown that $\int j\, d\theta$ is proportional to the slope of the dispersion for each branch, that is

$$ev_F \int (\psi_R^+\psi_R - \psi_L^+\psi_L)\, d\theta = \frac{e}{h}\frac{\partial\varepsilon(k)}{\partial k}$$



For the first subband, if one establishes a difference in the chemical potential between the left and the right dispersive branch,

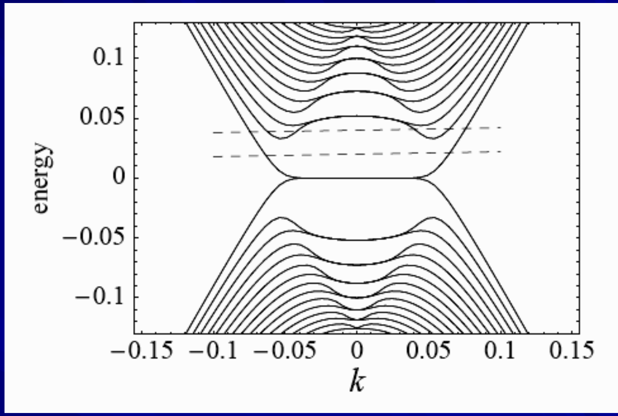
$$I = 4\int \frac{e}{h}\frac{\partial\varepsilon(k)}{\partial k} dk = 4\frac{e}{h}(\varepsilon_R - \varepsilon_L)$$

The Hall voltage, as would be measured in the flanks, is $V_H \approx (\varepsilon_R - \varepsilon_L)/e$ so that

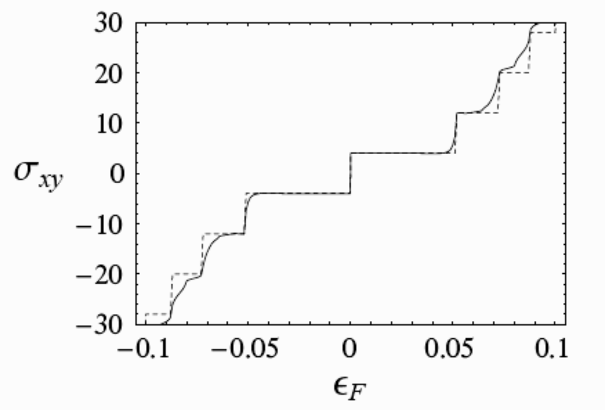
$$\sigma_{xy} = \frac{I}{V_H} \approx 4\frac{e^2}{h}$$

QUANTIZATION OF THE HALL CONDUCTIVITY

For the first subband the quantization of σ_{xy} has to be accurate, but for the next subbands it will depend on the profile of the Hall voltage accross the nanotube.



In the case of a linear Hall voltage drop around the nanotube, one may still observe steps in the measures of the Hall conductivity:



$$\sigma_{xy} \approx 2(2+4n)\frac{e^2}{h}$$

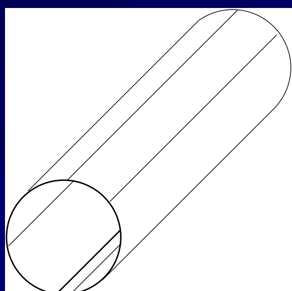
E. Perfetto *et al.*, Phys. Rev. B **76**, 125430 (2007)

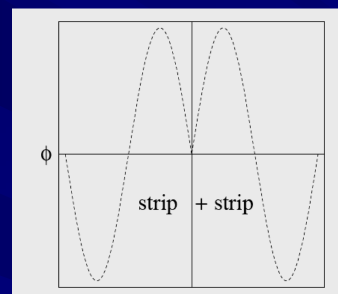
FROM CARBON NANOTUBES TO CURVED GRAPHENE STRIPS

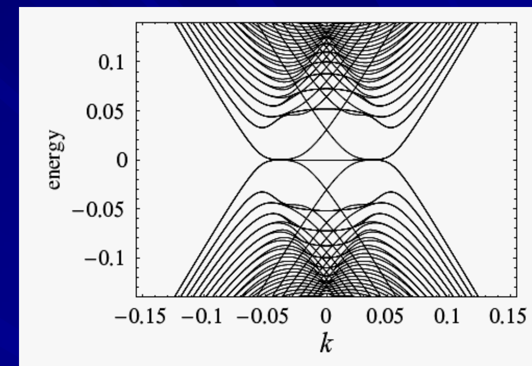
Quantization of the Hall conductivity: carbon nanotubes $\sigma_{xy} \approx 2(2+4n)\, e^2/h$

graphene $\sigma_{xy} \approx 2(1+2n)\, e^2/h$

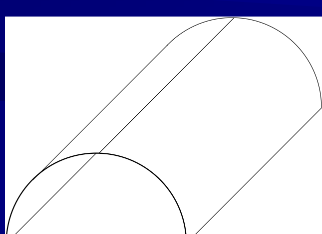
To understand the transition, we can cut longitudinally the carbon nanotube:

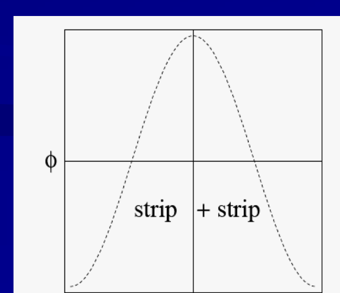


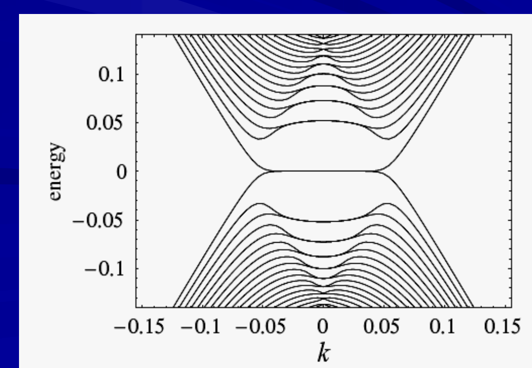




$$\Delta H_{p,p'} = -\frac{4}{\pi}\sum_m \delta_{p',p\pm 2m+1}\frac{1}{(2m+1)^2-4}\, v_F\, \frac{e}{c} BR\, \boldsymbol{\sigma}_3$$

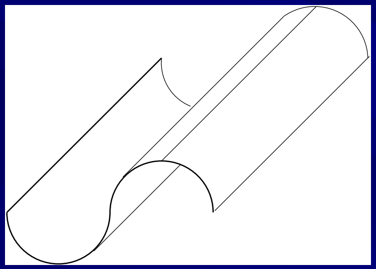


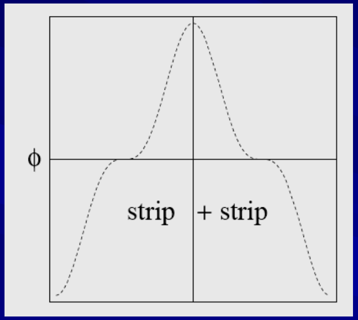


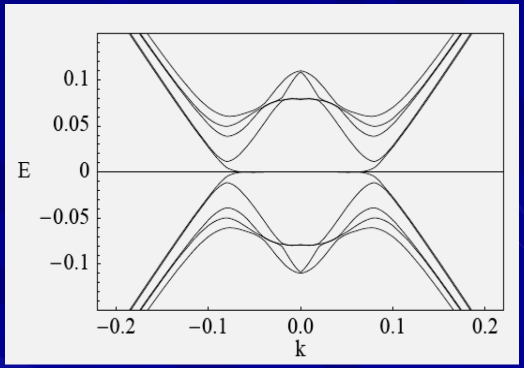


CORRUGATED GRAPHENE SURFACES

In general, the curvature of the graphene surface may lead to new dispersive branches and new edge states:







This shows that in general

- the degeneracy of the Landau subbands will depend on the topology of the carbon lattice

graphene $\sigma_{xy} \approx 2(1+2n)\, e^2/h$

carbon nanotubes $\sigma_{xy} \approx 2(2+4n)\, e^2/h$

- dispersive branches are expected from the regions where the normal component of the magnetic field vanishes, leading to different forms of "edge" states