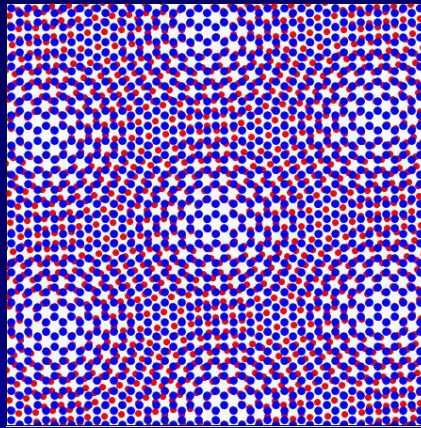


MARGINAL FERMI LIQUID AND DYNAMICAL SYMMETRY BREAKING FROM COULOMB INTERACTION IN TWISTED BILAYER GRAPHENE



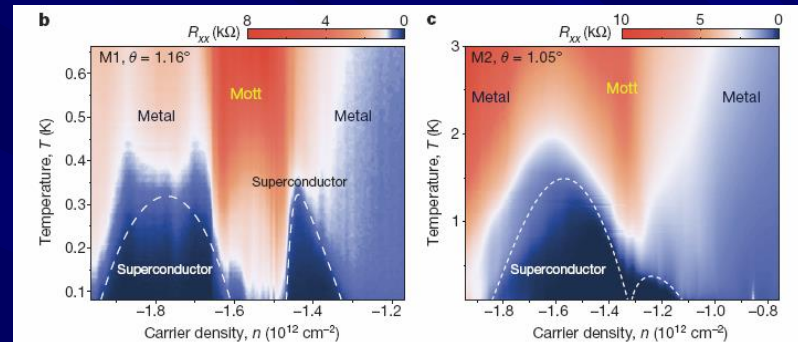
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¹Instituto de Estructura de la Materia, CSIC, Spain

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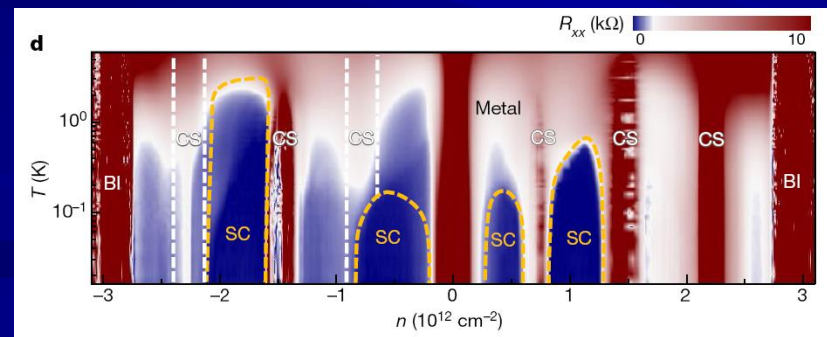
TWISTED BILAYER GRAPHENE

Magic-angle twisted bilayer graphene is a system with a very rich phase diagram, reflecting strong correlations at integer fillings



Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, *Nature* 556, 43 (2018)

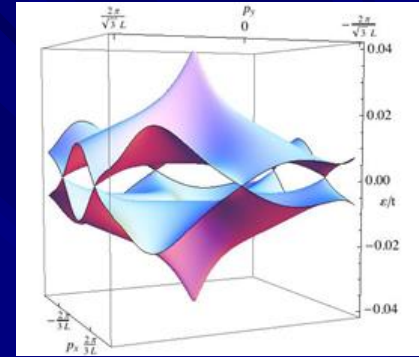
Insulating states have been observed at integer fillings of the first valence and conduction bands



X. Lu *et al.*, *Nature* 574, 653 (2019)

DYNAMICAL SYMMETRY BREAKING IN TWISTED BILAYER GRAPHENE

At the charge neutrality point, the opening of a gap can be understood as an effect of dynamical symmetry breaking at the Dirac cones of the noninteracting system.



Dynamical symmetry breaking can be studied by means of a self-consistent Hartree-Fock approximation. Starting from a tight-binding approach, the noninteracting Hamiltonian H_0 can be represented in the form

$$H_0 = \sum_a \varepsilon_a^0 \phi_a^0(\mathbf{r}_i) \phi_a^0(\mathbf{r}_j)^*$$

The Hartree-Fock approximation proceeds by assuming that the electron propagator G has a similar representation

$$G^{-1}(\mathbf{r}_i, \mathbf{r}_j) = G_0^{-1}(\mathbf{r}_i, \mathbf{r}_j) - \Sigma(\mathbf{r}_i, \mathbf{r}_j)$$

$$G_0(\mathbf{r}_i, \mathbf{r}_j) = -\sum_a \frac{1}{\varepsilon_a^0} \phi_a^0(\mathbf{r}_i) \phi_a^0(\mathbf{r}_j)^*$$

$$G(\mathbf{r}_i, \mathbf{r}_j) = -\sum_a \frac{1}{\varepsilon_a} \phi_a(\mathbf{r}_i) \phi_a(\mathbf{r}_j)^*$$

DYNAMICAL SYMMETRY BREAKING IN TWISTED BILAYER GRAPHENE

The condensation of the different order parameters can be studied through

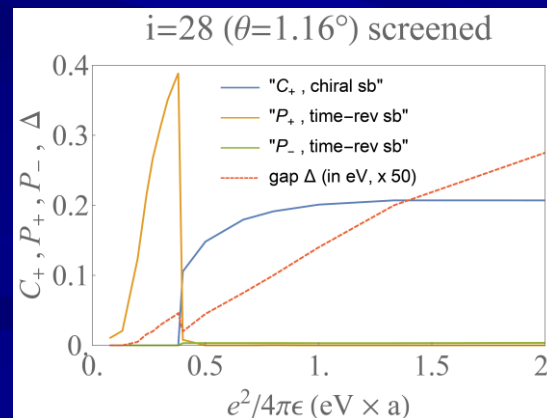
$$h_{ij} = \sum_{\substack{\text{filled} \\ \text{bands}}} \phi_a(\mathbf{r}_i) \phi_a(\mathbf{r}_j)^*$$

Chiral symmetry breaking, with charge asymmetry in sublattices A, B of layers 1, 2

$$C_{\pm} = \sum_{i \in A_1} h_{ii} - \sum_{i \in B_1} h_{ii} \pm \left(\sum_{i \in A_2} h_{ii} - \sum_{i \in B_2} h_{ii} \right)$$

Time-reversal symmetry breaking, with currents between nearest neighbors i_1, i_2, i_3 of each site

$$P_{\pm} = \text{Im} \left(\sum_{i \in A_1} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} + \sum_{i \in B_1} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} \pm \sum_{i \in A_2} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} \pm \sum_{i \in B_2} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} \right)$$



$$V(\mathbf{r}) = \frac{e^2}{4\pi\epsilon} 2\sqrt{2} \frac{e^{-\pi/\xi}}{\xi\sqrt{r/\xi}}$$

DYNAMICAL SYMMETRY BREAKING IN TWISTED BILAYER GRAPHENE

Chiral symmetry breaking, with charge asymmetry in sublattices A, B of layers 1, 2

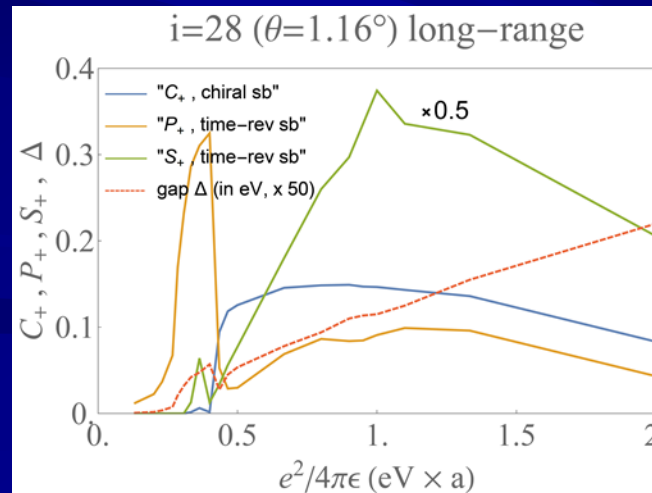
$$C_{\pm} = \sum_{i \in A_1} h_{ii} - \sum_{i \in B_1} h_{ii} \pm \left(\sum_{i \in A_2} h_{ii} - \sum_{i \in B_2} h_{ii} \right)$$

Time-reversal symmetry breaking, with currents between nearest neighbors i_1, i_2, i_3 of each site

$$P_{\pm} = \text{Im} \left(\sum_{i \in A_1} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} + \sum_{i \in B_1} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} \pm \sum_{i \in A_2} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} \pm \sum_{i \in B_2} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} \right)$$

In the case of unscreened $1/|\mathbf{r}|$ interaction, we have one more dominant order parameter

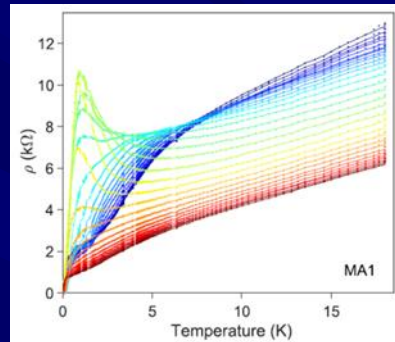
$$S_{+} = \text{Im} \left(\sum_{i \in A_1} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} - \sum_{i \in B_1} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} + \sum_{i \in A_2} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} - \sum_{i \in B_2} (h_{i_1 i_2} h_{i_2 i_3} h_{i_3 i_1})^{1/3} \right)$$



$$V(\mathbf{r}) = \frac{e^2}{4\pi\epsilon} \frac{1}{r}$$

MARGINAL FERMI LIQUID IN TWISTED BILAYER GRAPHENE

Near half-filling, T -linear resistivity is observed above the superconducting dome



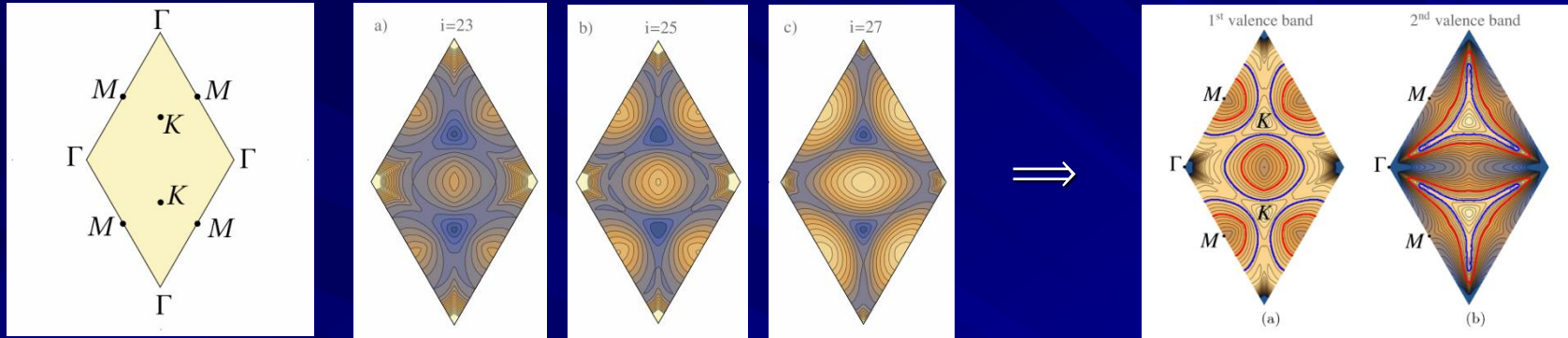
Y. Cao, D. Chowdhury, D. Rodan-Legrain, O. Rubies-Bigordà,
K. Watanabe, T. Taniguchi, T. Senthil, and P. Jarillo-Herrero,
Phys. Rev. Lett. 124, 076801 (2020)

This is a very challenging observation since electron-phonon scattering cannot account for such an effect at temperatures $T \sim 1$ K

$$\rho \propto T^5, \quad T \ll \Theta_D$$

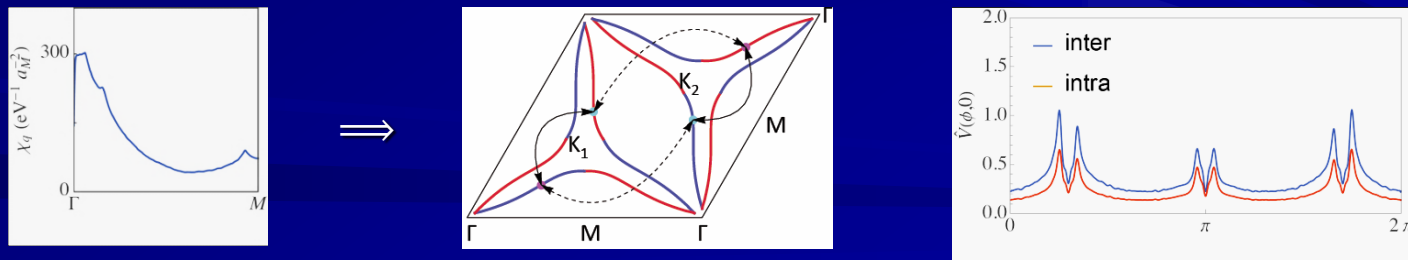
MARGINAL FERMI LIQUID IN TWISTED BILAYER GRAPHENE

We can turn to the effects of e - e scattering, which are rather strong given the special character of the Fermi line near the saddle points around the ΓK line



$$\cos(\theta_i) = \frac{3i^2 + 3i + 1/2}{3i^2 + 3i + 1}$$

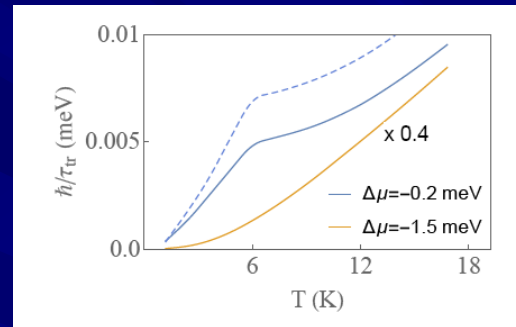
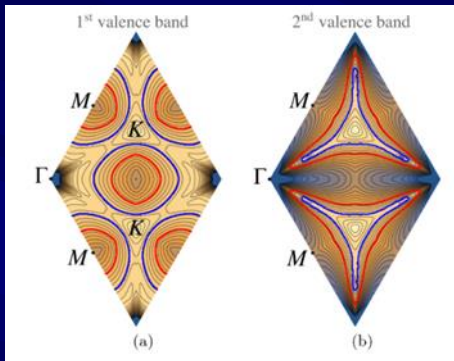
Near the magic angle, the straight segments of the Fermi line give rise to an enhanced electron-hole susceptibility. This accounts for the modulation needed in the Cooper-pair channel $V(\phi, \phi')$ to produce a superconducting instability



MARGINAL FERMI LIQUID IN TWISTED BILAYER GRAPHENE

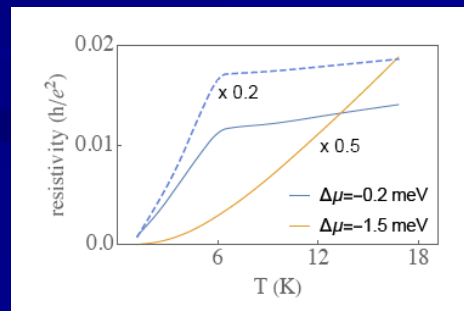
But the enhanced susceptibility has also important effects in the transport scattering rate

$$\frac{1}{\tau_{\text{tr}}(\mathbf{k})} = U^2 \int \frac{d^2 k'}{(2\pi)^2} \int_0^{\varepsilon_{\mathbf{k}}} d\omega \left| \langle \mathbf{k} | \mathbf{k}' \rangle \right|^2 (1 - n_F(\varepsilon_{\mathbf{k}'})) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'} - \omega) \text{Im} \chi_{\text{tr}}(\mathbf{k}, \mathbf{k}'; \omega)$$



The anomalous e - e scattering is translated to the resistivity

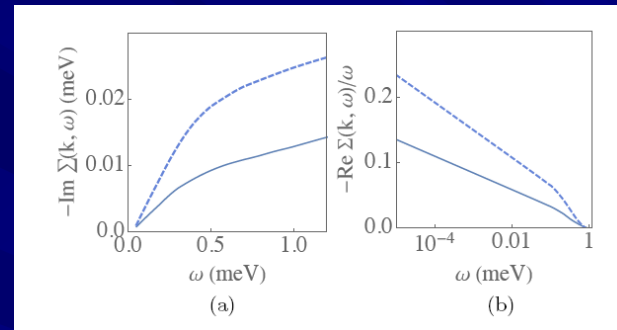
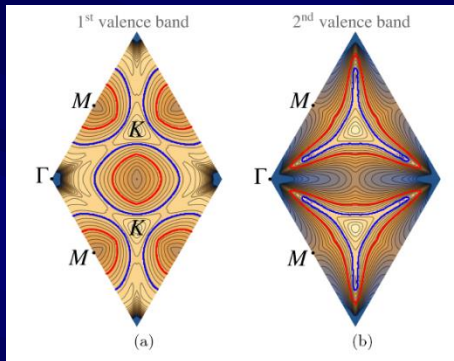
$$\rho_i \sim \rho_0 \frac{1}{T} \oint_{C_i} dk_{\parallel} \int d\varepsilon_{\mathbf{k}} \frac{1}{v_{\mathbf{k}}} \frac{n_F(\varepsilon_{\mathbf{k}})}{\tau_{\text{tr}}(\mathbf{k})}$$



MARGINAL FERMI LIQUID IN TWISTED BILAYER GRAPHENE

Not only the resistivity, but also quasiparticle properties display unconventional behavior

$$\text{Im} \Sigma(\mathbf{k}, \omega) = -U^2 \int \frac{d^2 p}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega_p \left| \langle \mathbf{k} | \mathbf{p} \rangle \right|^2 \text{sgn}(\omega_p) \delta(\omega_p - \varepsilon_p) \text{Im} \chi(\mathbf{k} - \mathbf{p}, \omega - \omega_p)$$



The anomalous transport properties characterize the so-called marginal Fermi liquid behavior, with a progressive attenuation of the quasiparticles at the Fermi level

$$G(\mathbf{k}, \omega) = \frac{1}{\omega - \varepsilon_{\mathbf{k}} - \Sigma(\mathbf{k}, \omega)} \sim \frac{1/|\log(\omega)|}{\omega - \varepsilon_{\mathbf{k}} + i\gamma \omega}$$

MARGINAL FERMI LIQUID IN TWISTED BILAYER GRAPHENE

$$G(\mathbf{k}, \omega) = \frac{1}{\omega - \varepsilon_{\mathbf{k}} - \Sigma(\mathbf{k}, \omega)} \sim \frac{1/|\log(\omega)|}{\omega - \varepsilon_{\mathbf{k}} + i\gamma \omega}$$

The marginal Fermi liquid behavior is also reflected in quantities like the entropy of the electron liquid

$$\begin{aligned} S &\sim \frac{1}{T} \int \frac{dk_{\parallel}}{v_{\mathbf{k}}} \int_{-\infty}^{\infty} d\omega \, \omega \frac{\partial n_F(\omega)}{\partial \omega} (\omega - \text{Re} \Sigma(\mathbf{k}, \omega)) \\ &\sim T |\log(T)| \end{aligned}$$

We end up with the prediction that a number of observables should be affected by the anomalous scaling

heat capacity	$C = T \frac{\partial}{\partial T} S \sim T \log(T) $
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thermal conductivity	$\kappa(T) = \alpha C \sim \log(T) $
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TWISTED BILAYER GRAPHENE

In conclusion,

- The Coulomb interaction has a very strong effective strength at the magic angle of twisted bilayer graphene, leading to effects which can be understood at the charge neutrality point in terms of dynamical symmetry breaking in Dirac-like systems
- The deviations from Fermi liquid behavior arising at half-filling of the first valence and conduction bands are more difficult to explain, and they point at an intense e - e scattering as a result of special features of the Fermi line
- We have seen that the special geometry of the Fermi line of the twisted bilayer near the magic angle plays an important role, with the potential to induce a pairing instability through the Kohn-Luttinger mechanism, as well as to lead to deviations from the conventional Fermi liquid picture

