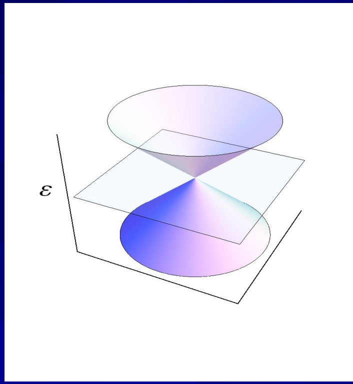
EFFECT OF DISORDER ON THE CRITICAL BEHAVIOR OF INTERACTING 3D DIRAC AND WEYL SEMIMETALS



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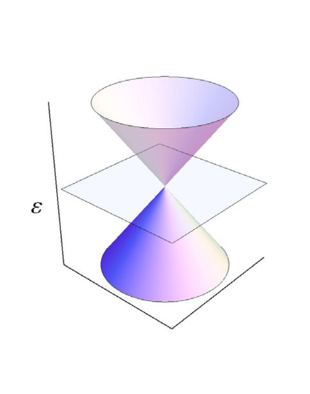
INTERACTING 3D SEMIMETALS

3D semimetals are electron systems which have to be modeled in terms of a set of Dirac or Weyl spinors with long-range Coulomb interaction



$$S = \int dt\, d^3x\;\; \Psi_i^+(\mathbf{x})\; \left(-i\partial_t - iv_F \gamma_0 \boldsymbol{\gamma}\cdot\partial\right)\; \Psi_i(\mathbf{x}) \\ -\; e_0 \int dt\, d^3x\;\; \Psi_i^+(\mathbf{x})\Psi_i(\mathbf{x})\;\; \phi(\mathbf{x})$$

For the scalar potential $\phi(\mathbf{x})$ we can take the free nonretarded propagator

$$D_0(\mathbf{q}, \omega_q) = \frac{1}{\mathbf{q}^2}$$

The vanishing density of states at the nodal points implies that the Coulomb potential remains long-ranged in the interacting theory

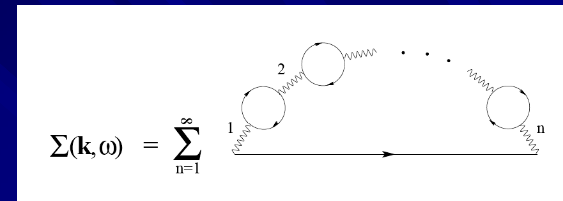
$$\lim_{\mathbf{q}\to 0} D(\mathbf{q},0)^{-1} \;=\; 0$$

which means that these semimetals may undergo important renormalization effects.

INTERACTING 3D SEMIMETALS

One may resort to a large-N approach to study the renormalization of the quasiparticle parameters. The electron propagator G is then given by

$$\frac{1}{G} = Z_\psi\left(\omega - Z_v v_R \gamma_0 \boldsymbol{\gamma}\cdot\mathbf{k}\right) - \Sigma$$



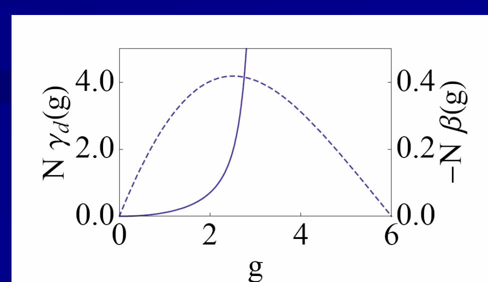
From the scale dependence of the renormalization factors we obtain two important functions

$$\frac{\mu}{Z_\psi}\frac{\partial Z_\psi}{\partial \mu} = \gamma(g) \qquad \frac{\mu}{Z_v}\frac{\partial Z_v}{\partial \mu} = -\beta(g) \quad , \qquad g \equiv \frac{Ne^2}{2\pi^2 v_R}$$

which correspond to the electron anomalous dimension and the scaling of the Fermi velocity

$$G(s\mathbf{k}, s\omega) = \frac{1}{s^{1-\gamma}}G(\mathbf{k},\omega) \qquad \frac{\mu}{v_R}\frac{\partial v_R}{\partial \mu} = \beta(g)$$

In the large-N limit, it is found that $\gamma(g)$ diverges at a critical coupling $g_c = 3$



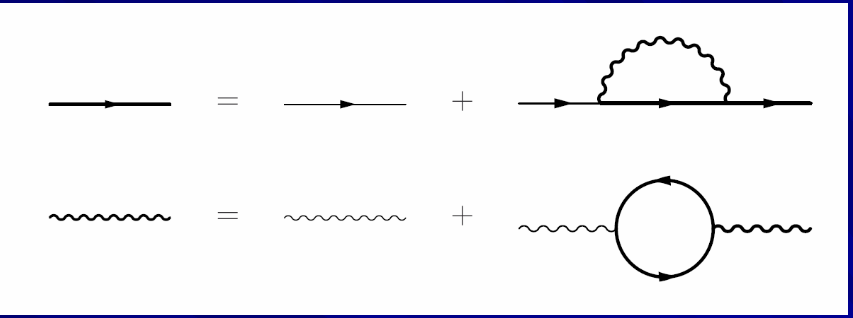
J.G., Phys. Rev. B **90**, 121107(R) (2014);
Phys. Rev. B **92**, 125115 (2015).

INTERACTING 3D SEMIMETALS

The critical behavior can be investigated by adopting a different nonperturbative approach, which consists in the resolution of the Schwinger-Dyson equations (bare vertex approximation):

$$G^{-1}(\mathbf{k},\omega) = \omega - v_F \gamma_0 \boldsymbol{\gamma}\cdot\mathbf{k} - \Sigma(\mathbf{k},\omega)$$

$$D^{-1}(\mathbf{q},\omega) = \frac{\mathbf{q}^2}{e^2} - \Pi(\mathbf{q},\omega)$$



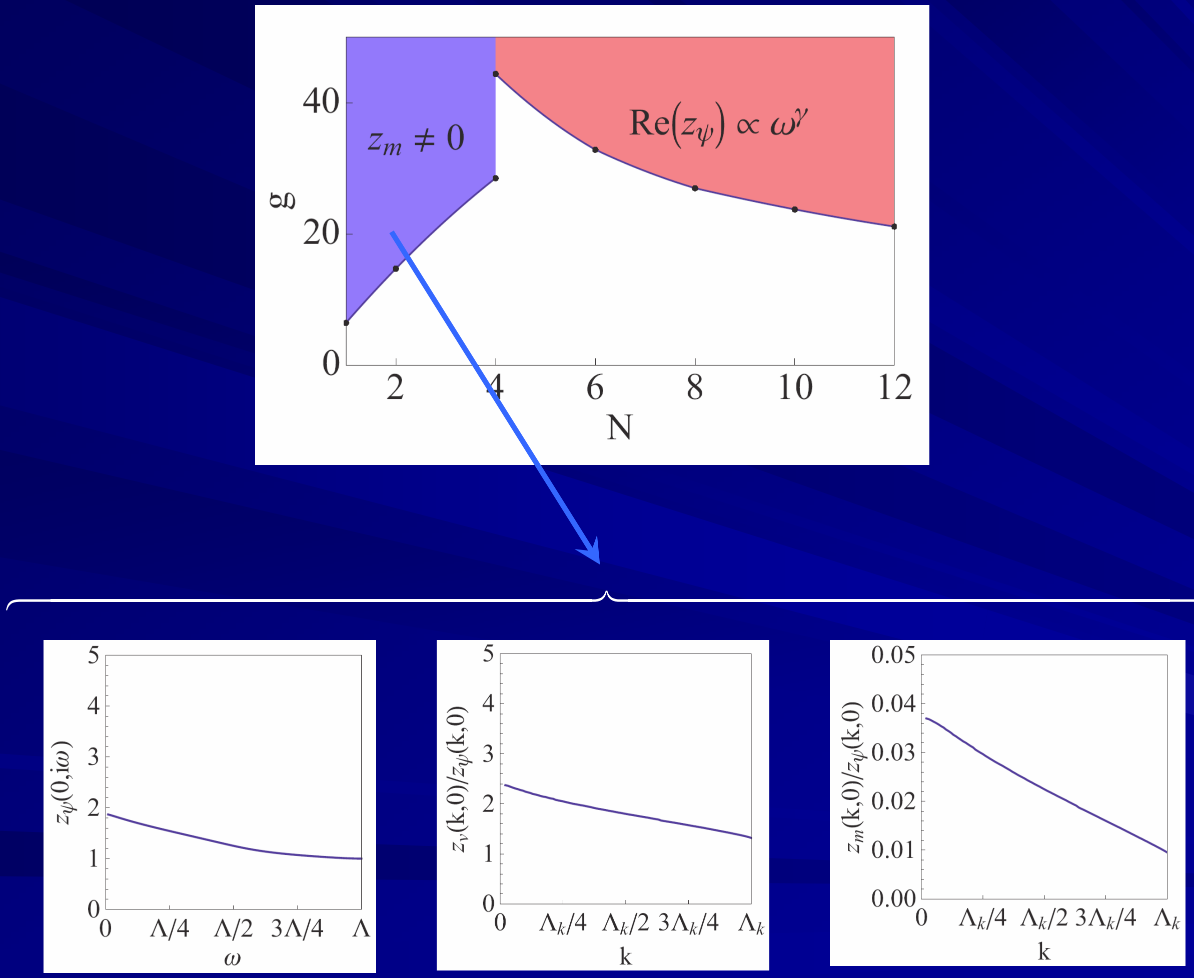
The self-consistent solution for the fermion propagator

$$G(\mathbf{k},\omega) = \frac{1}{z_\psi(\mathbf{k},\omega)\,\omega - z_v(\mathbf{k},\omega)\,v_F\gamma_0\boldsymbol{\gamma}\cdot\mathbf{k} + z_m(\mathbf{k},\omega)\gamma_0}$$

encodes then the renormalization of the quasiparticle parameters.

INTERACTING 3D SEMIMETALS

The phase diagram can be mapped in terms of the coupling g and the number N of nodes



$$g \equiv \frac{Ne^2}{4\pi v_F}$$

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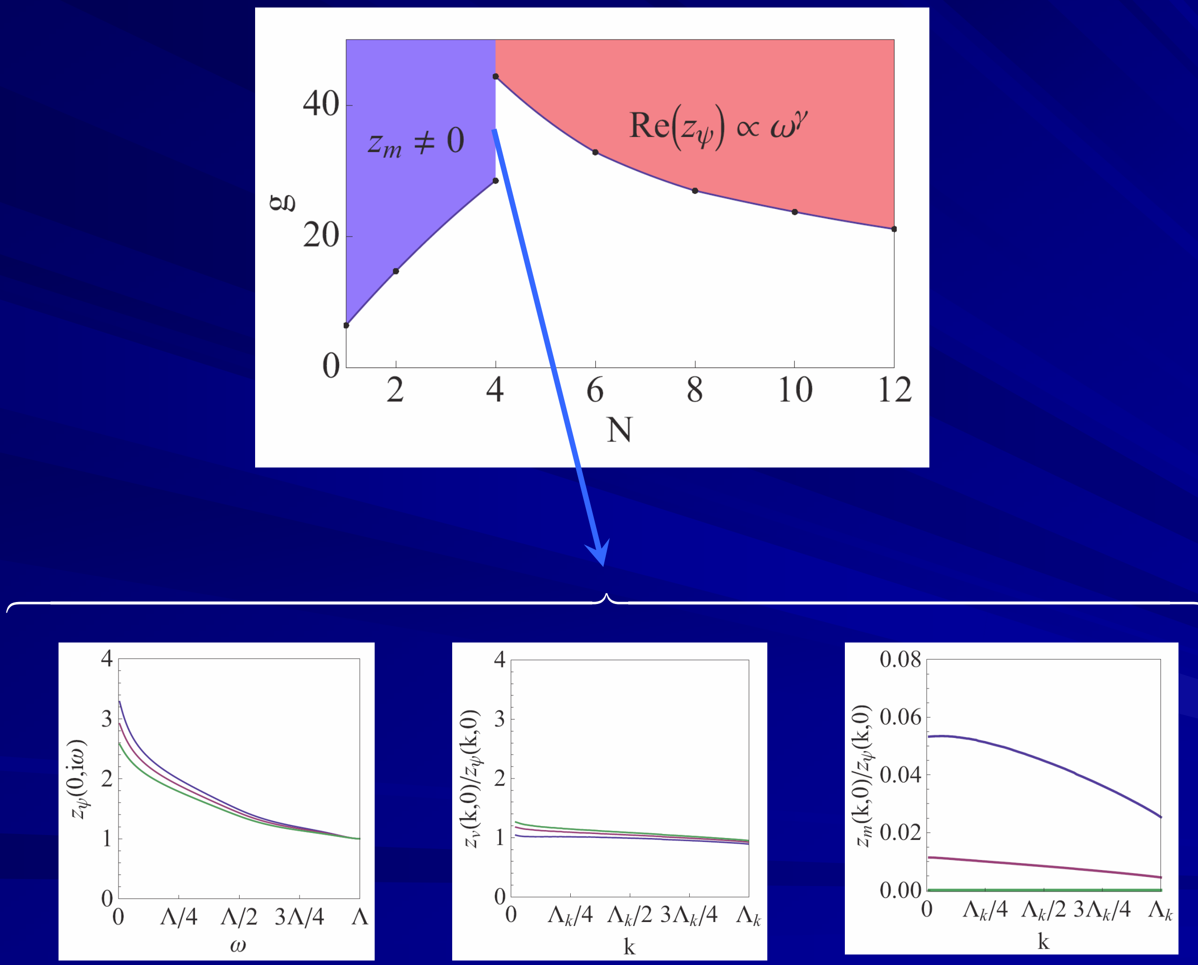

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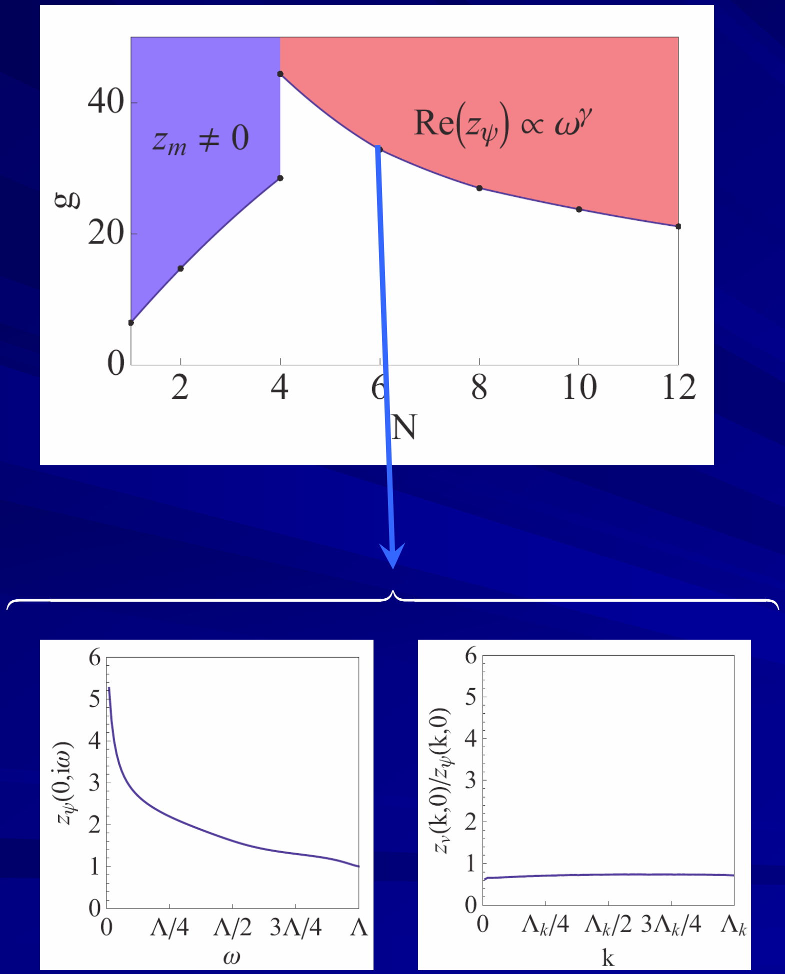

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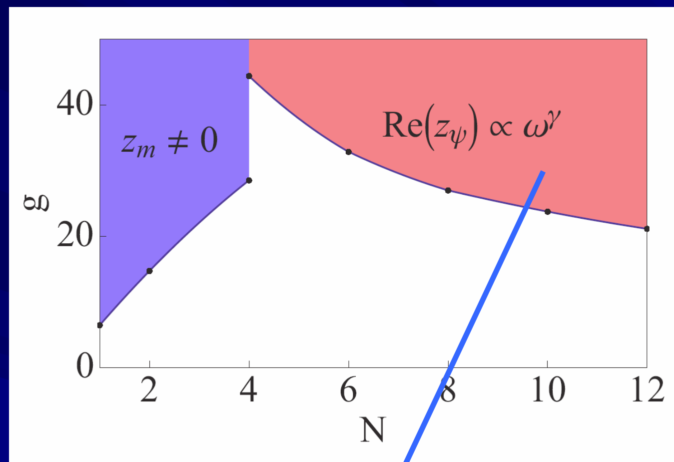

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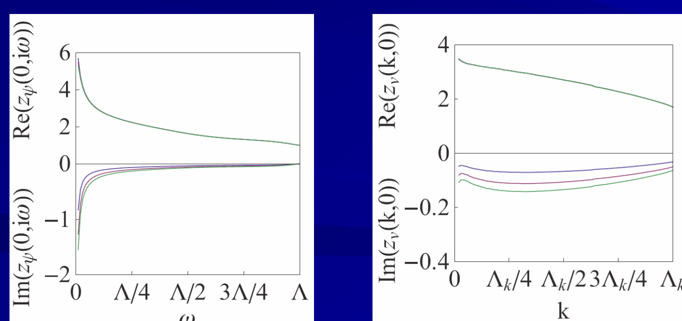

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DISORDERED 3D SEMIMETALS

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DISORDERED INTERACTING WEYL SEMIMETALS

We want to investigate the effects of disorder on the interacting theory

$$S = \int dt\, d^3x \;\; \Psi_i^+(\mathbf{x}) \;\left(-i\partial_t - iv_F \gamma_0 \boldsymbol{\gamma}\cdot\partial\right)\; \Psi_i\,(\mathbf{x}) \;-\; e_0 \int dt\, d^3x \;\; \Psi_i^+(\mathbf{x})\Psi_i\,(\mathbf{x})\, \phi(\mathbf{x})$$
$$- \int d^3x \;\; \Psi_i^+(\mathbf{x})\Psi_i\,(\mathbf{x}) \;\; \eta(\mathbf{x})$$

The disorder is represented by the random potential $\eta(\mathbf{x})$, which has in general a variance

$$\overline{\eta(\mathbf{x})\, \eta(\mathbf{x}')} = w(\mathbf{x} - \mathbf{x}')$$

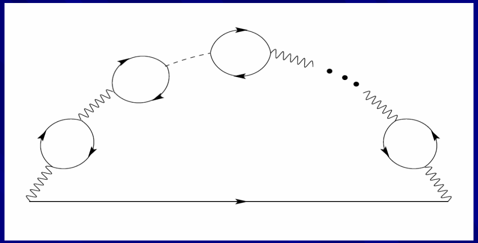
We choose the correlations of the disorder in such a way that the action remains scale invariant

$$w(\mathbf{x}) = \frac{w_0}{\mathbf{x}^2}$$

though it can be checked that this choice does not modify qualitatively the phases with respect to the case with uncorrelated disorder.

DISORDERED INTERACTING WEYL SEMIMETALS

By using the replica method we can compute again the electron self-energy at large *N*, which gets corrections from the correlation of the disorder potential



$$\frac{1}{G} = Z_\psi\left(\omega - Z_v v_R \gamma_0 \boldsymbol{\gamma}\cdot\mathbf{k}\right) - \Sigma$$

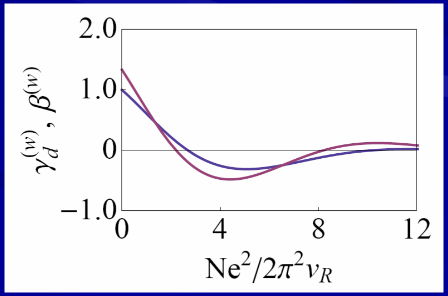
An important feature is that the strength of the disorder needs to be renormalized together with the value of the electron charge, leading to the low-energy scaling

$$e^2(\mu) = \frac{e_0^2}{1 + \frac{N e_0^2}{6\pi^2 v_F}\log(\Lambda/\mu)} \quad , \quad w_R(\mu) = \frac{w_0}{\left(1 + \frac{N e_0^2}{6\pi^2 v_F}\log(\Lambda/\mu)\right)^2}$$

Besides, we get corrections to the anomalous dimension and the scaling of the Fermi velocity

$$\gamma(g) = \frac{1}{N}\gamma^{(0)}(g) + \frac{w_R}{v_R^2}\gamma^{(w)}(g) + O\left(w_R^2\right)$$

$$\frac{\mu}{v_R}\frac{\partial v_R}{\partial \mu} = \frac{1}{N}\beta^{(0)}(g) + \frac{w_R}{v_R^2}\beta^{(w)}(g) + O\left(w_R^2\right)$$



J.G., arXiv:1703.02885

DISORDERED INTERACTING WEYL SEMIMETALS

We get therefore the competition between two different effects. On one hand we have the screening of the interactions, which is encoded in the large-N equations

$$\mu \frac{\partial}{\partial \mu} e^2 = \frac{N e^4}{6\pi^2 v_R} \quad , \quad \mu \frac{\partial}{\partial \mu} w_R = \frac{N\, w_R e^2}{3\pi^2 v_R}$$

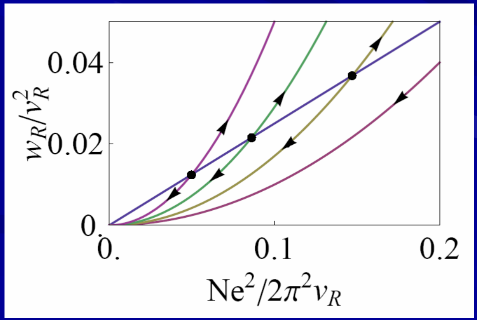
On the other hand, we have the low-energy scaling of v_R, driven to lower values by the disorder

$$\mu \frac{\partial\, v_R}{\partial \mu} = -\frac{1}{N} \frac{N e^2}{6\pi^2 v_R} + \frac{4}{3} \frac{w_R}{v_R^2} + \ldots$$

The effective strengths of the Coulomb interaction $g = Ne^2/2\pi^2 v_R$ and the disorder $w_R/(v_R)^2$ obey the equations to quadratic order

$$\mu \frac{\partial}{\partial \mu} g = \frac{1}{3} g^2 - \frac{4}{3} \frac{w_R}{v_R^2} g$$

$$\mu \frac{\partial}{\partial \mu} \frac{w_R}{v_R^2} = \frac{2}{3} \frac{w_R}{v_R^2} g - \frac{8}{3} \left(\frac{w_R}{v_R^2} \right)^2$$



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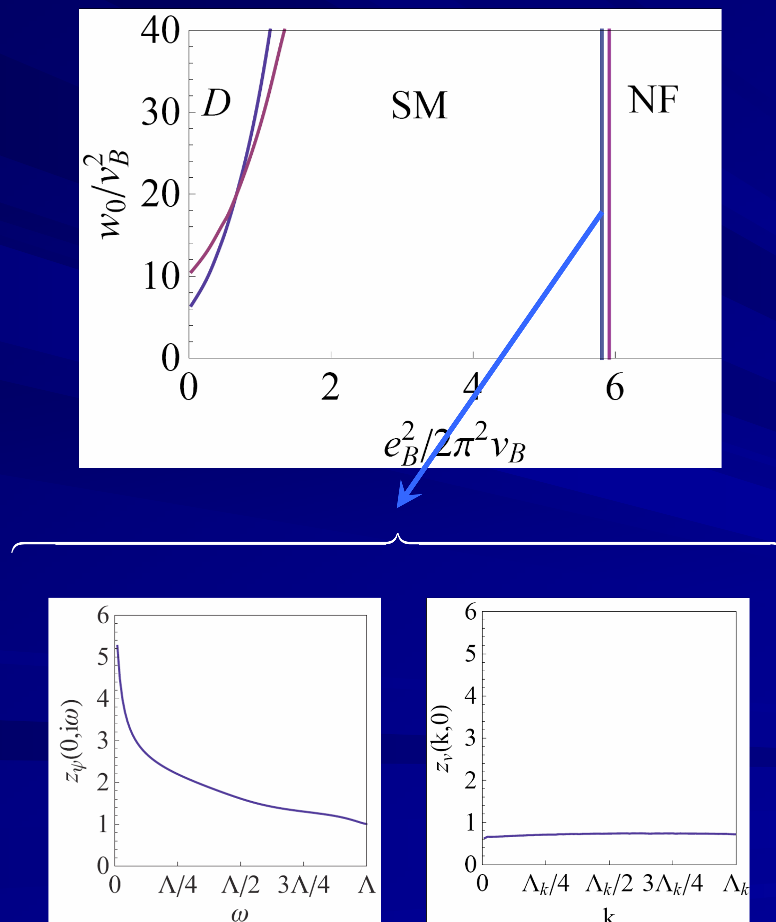
DISORDERED INTERACTING WEYL SEMIMETALS

Finally, we can accomplish the self-consistent resolution of the Schwinger-Dyson equations in the presence of disorder, introducing a Weyl propagator

$$G(\mathbf{k},\omega) = \left[z_\psi(\mathbf{k},\omega) \; \left(\omega \; - \; z_v(\mathbf{k},\omega)\, v_F \gamma_0 \boldsymbol{\gamma}\cdot\mathbf{k}\right) \right]^{-1}$$



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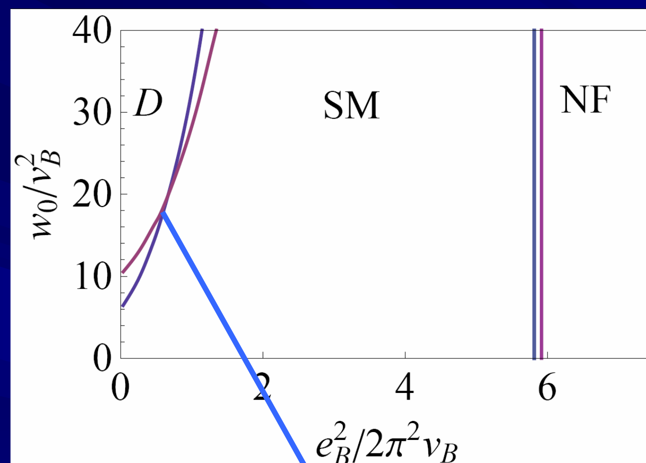

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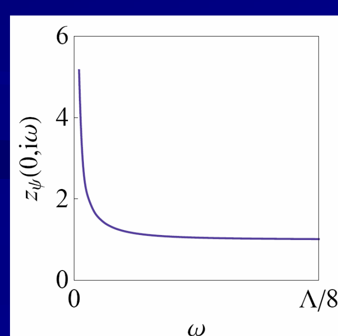

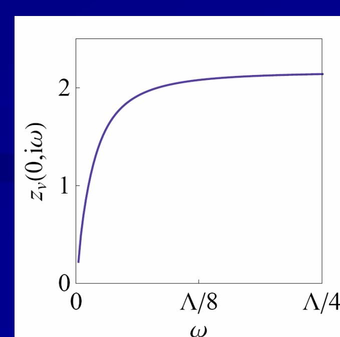

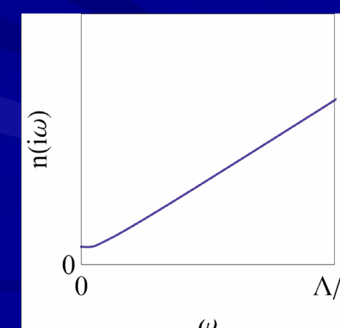

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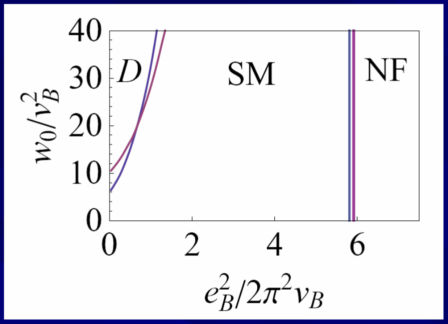
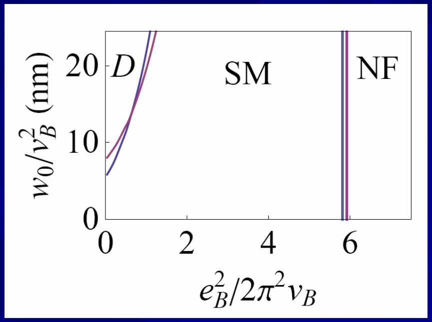
DISORDERED INTERACTING WEYL SEMIMETALS

In conclusion,

- we have seen that the phase diagram of disordered interacting Weyl semimetals contains three different phases, which are present no matter the specific choice of short-ranged correlation



$$\overline{\eta(\mathbf{x})\,\eta(\mathbf{x}')} = w_0\,/(\mathbf{x}-\mathbf{x}')^2$$



$$\overline{\eta(\mathbf{x})\,\eta(\mathbf{x}')} = w_0\;\delta(\mathbf{x}-\mathbf{x}')$$

- the phase induced by disorder is confined to the regime of weak interaction, while typical 3D Weyl semimetals have $v_F \lesssim 1$ eV nm and $e^2/2\pi^2 v_F \gtrsim 1$, leading to the possibility to observe the non-Fermi liquid phase in real materials

J.G., arXiv:1703.02885