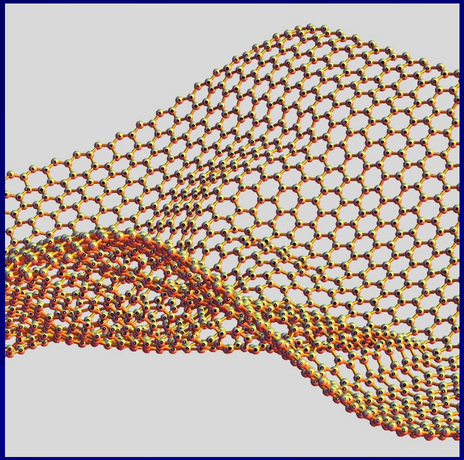
HIGGS-LIKE CONDENSATION OF RIPPLES AND BUCKLING TRANSITION IN GRAPHENE



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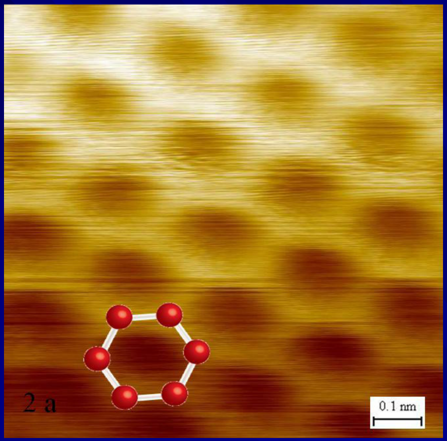
GRAPHENE

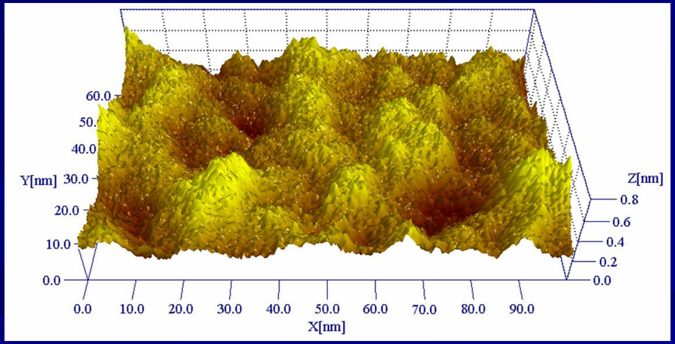
Graphene offers the possibility to study the behavior of electrons in a genuine two-dimensional system:

- access to the physics of relativistic massless fermions in $D = 2$
- great potential for applications from very large electron mobilities, flexibility and extreme mechanical strength



From E. Stolyarova *et al.*,

Proc. Natl. Acad. Sci. 104, 9209 (2007)



From E. Stolyarova *et al.*,

Proc. Natl. Acad. Sci. 104, 9209 (2007)

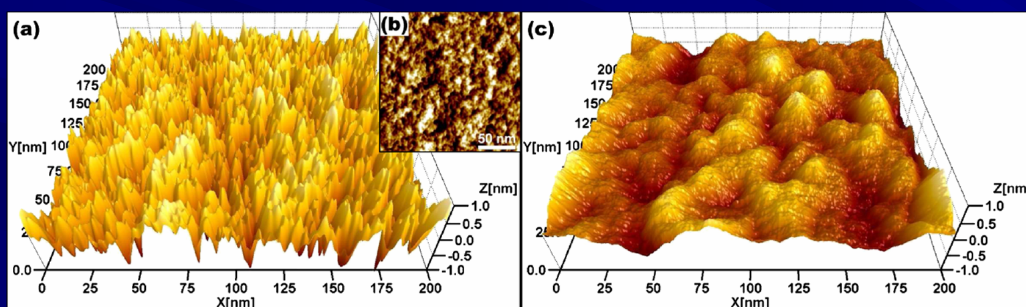
But some challenges have to be faced:

- the interaction with the substrate and boundary conditions modify significantly the transport properties
- samples have significant corrugation

RIPPLES IN GRAPHENE

In exfoliated graphene, ripples are correlated to some extent with the corrugation of the substrate, but they also arise in part as an effect intrinsic to the two-dimensional membrane:

- J. Meyer, A. Geim, M. Katsnelson, K. Novoselov, T. Booth and S. Roth, Nature 446, 60 (2007).
- W. Bao, F. Miao, Z. Chen, H. Zhang, W. Jang, C. Dames and C. N. Lau, Nature Nanotech. 4, 562 (2009).
- V. Geringer, M. Liebmann, T. Echtermeyer, S. Runte, M. Schmidt, R. Rückamp, M. C. Lemme, and M. Morgenstern, Phys. Rev. Lett. 102, 076102 (2009).



From V. Geringer *et al.*, PRL 102, 076102 (2007)

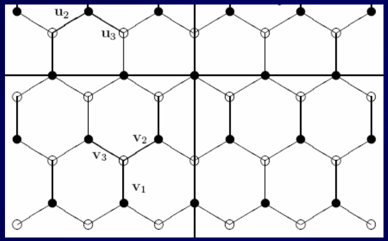
There have been a few theoretical proposals to understand the existence of ripples in graphene, either from Monte Carlo simulations of the membrane

- A. Fasolino, J. H. Los, and M. I. Katsnelson, Nature Mater. 6, 858 (2007)

or from the behavior as an electronic membrane (E.-A. Kim and A. H. Castro Neto, EPL 84, 57007 (2008))

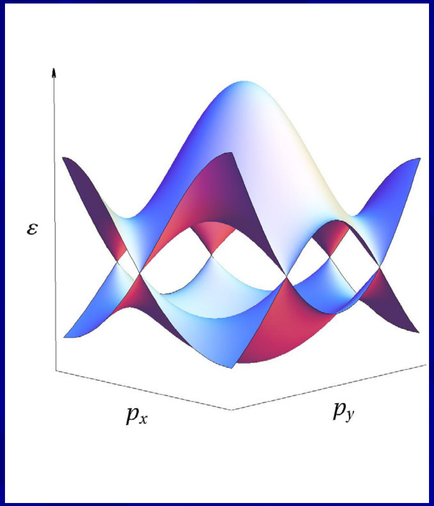
- D. Gazit, Phys. Rev. B 80, 161406(R) (2009)
- P. San-José, J. G. and F. Guinea, Phys. Rev. Lett. 106, 045502 (2011)

ELECTRONIC PROPERTIES OF GRAPHENE



The observed properties were actually consistent with the dispersion expected for electrons in a honeycomb lattice

$$H_{tb} = -t \sum_{r,r'} \psi^{+}(\mathbf{r}') \ \psi(\mathbf{r}) \longrightarrow H = -t \begin{pmatrix} 0 & \sum_a e^{i\mathbf{p}\cdot v_a} \\ \sum_a e^{-i\mathbf{p}\cdot v_a} & 0 \end{pmatrix}$$



$$E = \pm t \sqrt{1 + 4\cos^2(ap_y/2) + 4\cos(ap_y/2)\cos(\sqrt{3}ap_x/2)}$$

Expanding around each corner of the Brillouin zone, we obtain the hamiltonian for a two-component fermion (Dirac hamiltonian)

$$H = v_F \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

We have to introduce a Dirac fermion for each independent Fermi point, at which

$$H = v_F \boldsymbol{\sigma} \cdot \mathbf{p} \qquad , \qquad \varepsilon(\mathbf{p}) = \pm v_F |\mathbf{p}|$$

MANY-BODY EFFECTS IN GRAPHENE

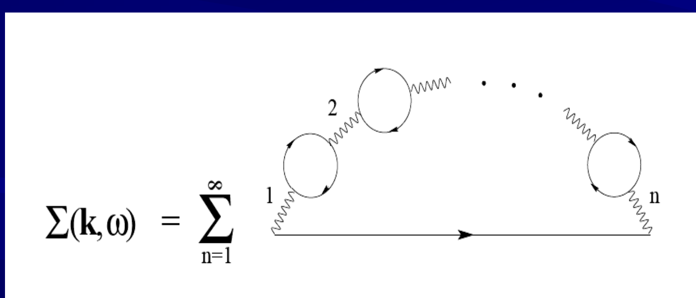
Graphene is a system with remarkable many-body properties, which can be traced back to the fact that the action of the interacting theory

$$S = \int dt\, d^2x\;\; \Psi_\sigma^{(a)+}(\mathbf{x})\;\; i\left(\partial_t - v_F \boldsymbol{\sigma}^{(a)} \cdot \partial\right)\, \Psi_\sigma^{(a)}(\mathbf{x}) \;-\; e^2 \int dt\, d^2x\, d^2x'\;\; \Psi_\sigma^{(a)+}(\mathbf{x})\, \Psi_\sigma^{(a)}(\mathbf{x})\, \frac{1}{|\mathbf{x}-\mathbf{x}'|}\, \Psi_{\sigma'}^{(b)+}(\mathbf{x}')\, \Psi_{\sigma'}^{(b)}(\mathbf{x}')$$

is invariant under the scale transformation

$$t \to st \quad , \quad \mathbf{x} \to s\mathbf{x} \quad , \quad \Psi(\mathbf{x}) \to s^{-1}\, \Psi(\mathbf{x})$$

This leads to the logarithmic scaling of many observables driven by the Coulomb interaction:



$$\frac{1}{G} = \frac{1}{G_0} - \Sigma \approx \omega - v_F \boldsymbol{\sigma}\cdot\mathbf{k}$$

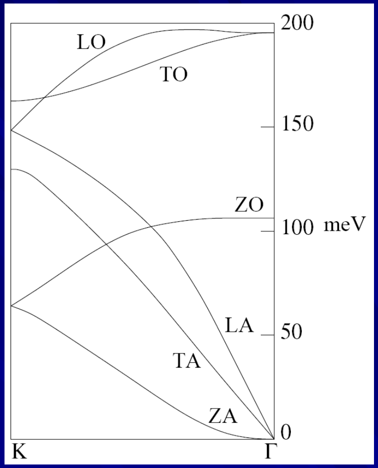
$$- \gamma(g)\; \left(\omega - \beta(g) v_F \boldsymbol{\sigma}\cdot\mathbf{k}\right)\, \log(E_c/\omega)$$

$$(g \equiv e^2/16 v_F)$$

(J. G., F. Guinea and M. A. H. Vozmediano, Nucl . Phys. B424, 595 (1994), Phys. Rev. B 59, 2474 (1999))

ELASTIC PROPERTIES OF GRAPHENE

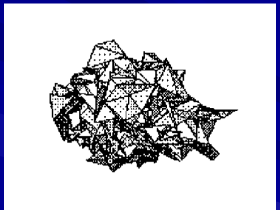
Graphene has also phonon modes, which confer the elastic properties to the material. The lowest-energy phonon branch correspond to flexural phonons, which must have at low momenta the dispersion



$$\varepsilon(\mathbf{p}) \propto \mathbf{p}^2$$

It is a nontrivial fact that a two-dimensional membrane can exist with such massless modes. Two important issues are

- stability of the membrane against crumpling (development of a phase with uncorrelated normals at the surface) →



- stability of the flat phase of the membrane against spontaneous symmetry breaking (development of an inhomogeneous buckled surface) →

ELASTIC PROPERTIES OF MEMBRANES

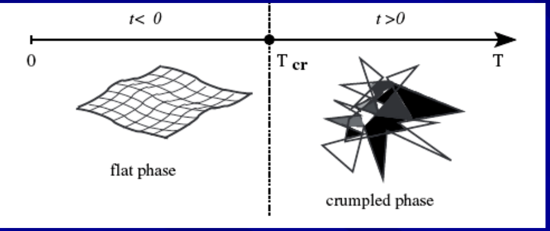
D-dimensional membranes $\mathbf{r}(\mathbf{x})$ embedded in a *d*-dimensional ambient space have in general a free energy

$$F = \int d^D x \left[\frac{\kappa}{2} (\partial^2 \mathbf{r})^2 + u(\partial_i \mathbf{r} \partial_j \mathbf{r})(\partial_i \mathbf{r} \partial_j \mathbf{r}) + v(\partial_i \mathbf{r} \partial_i \mathbf{r})^2 + \frac{t}{2} (\partial_i \mathbf{r} \partial_i \mathbf{r}) \right]$$

For membranes with fixed connectivity there is a preferred frame in which we can decompose

$$\mathbf{r}(\mathbf{x}) = \left(\xi\, \mathbf{x} + \mathbf{u}(\mathbf{x}), \mathbf{h}(\mathbf{x}) \right)$$

ξ is an order parameter for the crumpling transition. The fixed-connectivity membranes are shown to have a critical temperature T_c, below which $\xi \neq 0$ and



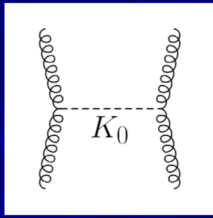
$$F = \frac{1}{2} \underbrace{\kappa \int d^D x \left(\partial^2 h \right)^2}_{\text{bending}} + \frac{1}{2} \underbrace{\int d^D x \left(2\mu\, u_{ij} u_{ij} + \lambda\, u_{ii} u_{jj} \right)}_{\text{stretching}} + \frac{1}{2} t \underbrace{\int d^D x\, u_{ii}}_{\text{tension}}$$

in terms of the strain tensor $$u_{ij} = \frac{1}{2} \left(\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h \right)$$

ELASTIC PROPERTIES OF MEMBRANES

To understand the flat phase at low temperature, we may integrate out the in-plane phonon modes (D. R. Nelson and L. Peliti, J. Physique **48**, 1085 (1987)):

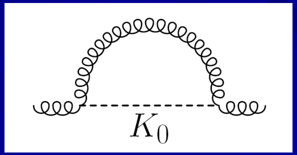
$$F = \frac{1}{2}\kappa \int d^2x\, (\partial^2 h)^2 + \frac{1}{2} K_0 \int d^2x \left(\frac{1}{2} P_{ij}^T \partial_i h \partial_j h \right)^2 \quad , \quad K_0 = 2\mu + \lambda - \frac{\lambda^2}{2\mu + \lambda}$$



This is now an interacting theory in which the flexural phonons get self-energy corrections

$$(\varepsilon(\mathbf{p}))^2 = \kappa_0 \mathbf{p}^4 + \Sigma(\mathbf{p})$$

$$\kappa\, \mathbf{p}^4 \approx \kappa_0 \mathbf{p}^4 + k_B T\, K_0\, \mathbf{p}^4 \int \frac{d^2q}{(2\pi)^2} \left(1 - \frac{(\mathbf{q}\cdot\mathbf{e}_p)^2}{\mathbf{q}^2} \right)^2 \frac{1}{\kappa\, (\mathbf{p}+\mathbf{q})^4}$$



The self-consistent solution of the above equation leads to

$$\kappa_{\rm eff}(\mathbf{p}) \sim \sqrt{k_B T\, K_0}\, \frac{1}{|\mathbf{p}|} \qquad \Rightarrow \qquad \langle \theta^2(\mathbf{x}) \rangle \approx \langle (\partial h)^2 \rangle = k_B T \int \frac{d^2q}{(2\pi)^2} \frac{1}{\kappa_{\rm eff}(\mathbf{q})\, \mathbf{q}^2} < \infty$$

Further refinement in the computation of $\kappa_{\rm eff}(p) \sim p^{-\eta}$ seems to converge to an anomalous exponent $\eta \approx 0.8$ (P. Le Doussal and L. Radzihovsky, PRL **69**, 1209 (1992)).

ELASTIC PROPERTIES OF GRAPHENE

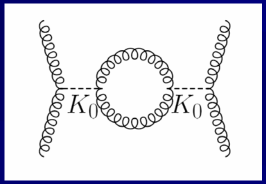
Graphene has a relatively large stiffness, which makes relevant the zero-temperature limit:

$$S = \frac{1}{2}\int dt\, d^2x \left(\rho(\partial_t h(\mathbf{x}))^2 - \kappa(\partial^2 h(\mathbf{x}))^2\right) - \frac{1}{2} K_0 \int dt\, d^2x \left(\frac{1}{2} P_{ij}^T \partial_i h(\mathbf{x})\, \partial_j h(\mathbf{x})\right)^2$$

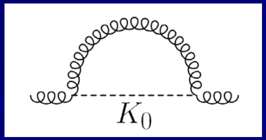
The theory for the flexural modes is invariant at the classical level under the transformation

$$t \to s^2 t \quad , \quad \mathbf{x} \to s\mathbf{x} \quad , \quad h(\mathbf{x}) \to h(\mathbf{x})$$

This opens the way to study the scaling of the different parameters in the quantum theory



$$\sim \frac{K_0^2}{\rho^2} \int d\omega\, d^2q \frac{q^4}{(\omega^2 - \varepsilon^2(\mathbf{q}))^2} \sim \frac{K_0^2}{\rho^2} \int d^2q \frac{q^4}{\varepsilon(\mathbf{q})^3} \sim \frac{K_0^2}{\sqrt{\rho}\kappa^{3/2}} \int^{q_c} dq \frac{1}{|\mathbf{q}|}$$

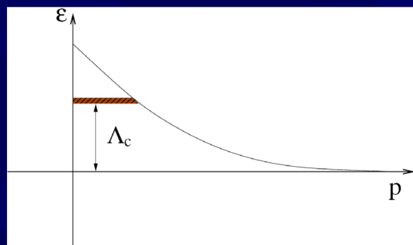


$$\sim \mathbf{p}^4 \frac{K_0}{\rho} \int d\omega\, d^2q \frac{1}{\omega^2 - \varepsilon^2(\mathbf{q})} \sim \mathbf{p}^4 \frac{K_0}{\rho} \int d^2q \frac{1}{\varepsilon(\mathbf{q})} \sim \mathbf{p}^4 \frac{K_0}{\sqrt{\rho\kappa}} \int^{q_c} dq \frac{1}{|\mathbf{q}|}$$

which is consistent with a steady renormalization of the interaction coupling towards lower values in the infrared.

ELASTIC PROPERTIES OF GRAPHENE

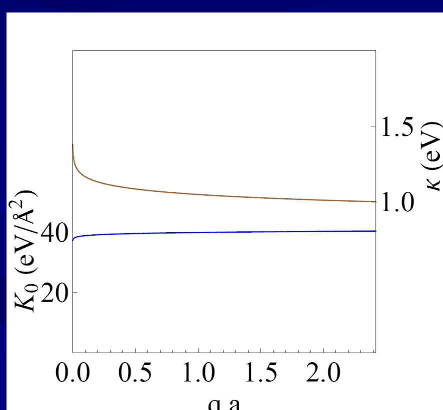
In a wilsonian renormalization group approach, we get upon progressive integration of high-energy shells:



$$q\frac{\partial K_0}{\partial q} = \frac{3}{64\pi}\frac{K_0^2}{\sqrt{\rho\kappa^{3/2}}}$$

$$q\frac{\partial \kappa}{\partial q} = -\frac{3}{16\pi}\frac{K_0}{\sqrt{\rho\kappa}}$$

We obtain the counterpart of the behavior found in the statistical field theory of membranes, now extended to the quantum theory at $T = 0$:



- the membrane (without conduction electrons) should flow at large distances to the weak coupling regime, with asymptotic behavior

$$K_0 \sim \frac{1}{\left(\log(q_c / q)\right)^{1/7}}$$

$$\kappa \sim \left(\log(q_c / q)\right)^{4/7}$$

ELASTIC PROPERTIES OF GRAPHENE

But in graphene there is also a strong coupling of the electrons to the phonon modes, which takes place mainly through charge density fluctuations

$$S_{\text{e-ph}} = g \int dt\, d^2x\, \Psi^+(\mathbf{x})\Psi(\mathbf{x}) \left(2\,\partial_i u_i(\mathbf{x}) + \partial_i h(\mathbf{x})\partial_i h(\mathbf{x})\right)$$

We can inspect the scaling of this interaction from the point of view of the flexural phonons:

$$S = \frac{1}{2}\int dt\, d^2x \left(\rho(\partial_t h(\mathbf{x}))^2 - \kappa(\partial^2 h(\mathbf{x}))^2\right) - \frac{1}{2}K_0 \int dt\, d^2x \left(\frac{1}{2}P_{ij}^T \partial_i h(\mathbf{x})\,\partial_j h(\mathbf{x})\right)^2$$

$$t \to s^2 t \quad , \quad \mathbf{x} \to s\mathbf{x} \quad , \quad h(\mathbf{x}) \to h(\mathbf{x})$$

This leads to a modified scaling of the Dirac fermions:

$$S = \int dt\, d^2x\;\; \Psi_\sigma^{(a)+}(\mathbf{x})\;\; i\left(\partial_t - v_F \boldsymbol{\sigma}^{(a)} \cdot \partial\right)\; \Psi_\sigma^{(a)}(\mathbf{x})$$

$$t \to s^2 t \quad , \quad \mathbf{x} \to s\mathbf{x} \quad , \quad \Psi(\mathbf{x}) \to s^{-3/2}\,\Psi(\mathbf{x})$$

under which the electron-phonon interaction turns out to be irrelevant at large-distance scales ($s \to \infty$)

$$S_{\text{e-ph}} = g \int \underbrace{dt d^2x}_{s^4}\; \underbrace{\Psi^+(\mathbf{x})\Psi(\mathbf{x})}_{\frac{1}{s^3}}\; \underbrace{\left(2\,\partial_i u_i(\mathbf{x}) + \partial_i h(\mathbf{x})\partial_i h(\mathbf{x})\right)}_{\frac{1}{s^2}}$$

ELASTIC PROPERTIES OF GRAPHENE

However, the electron-phonon coupling gives rise to an effective attractive interaction which may drastically modify the behavior of the flexural phonons.

We can integrate first the electron degrees of freedom:

$$S_{\text{u-u}} = 2g^2 \int d^2q \, d\omega \, \chi(\mathbf{q},0) \; u_{ii} u_{jj} \quad , \quad \chi(\mathbf{q},\omega) = -\frac{\mathbf{q}^2}{4\sqrt{v_F^2 \mathbf{q}^2 - \omega^2}}$$

We observe that the effective interaction acts as a shift in the Lamé constant

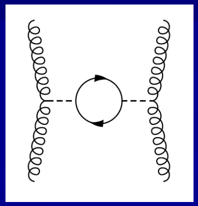
$$\lambda \to \lambda - g^2 \frac{|\mathbf{q}|}{v_F}$$

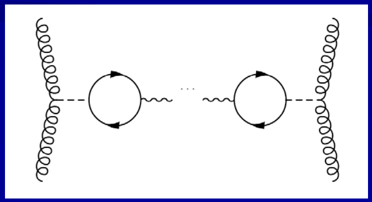
Integrating then the in-plane phonons as before, we get the action for flexural phonons

$$S = \frac{1}{2}\int dt \, d^2x \left(\rho(\partial_t h)^2 - \kappa(\partial^2 h)^2\right) - \frac{1}{2}\int dt \, d^2x d^2x' \left(\frac{1}{2} P_{ij}^T \partial_i h \partial_j h\right)_x K_{x,x'} \left(\frac{1}{2} P_{ij}^T \partial_i h \partial_j h\right)_{x'}$$

with interaction potential

$$K(\mathbf{q}) = 2\mu + \lambda - \underbrace{g^2 \frac{|\mathbf{q}|}{v_F}} - \underbrace{\frac{(\lambda - g^2|\mathbf{q}|/v_F)^2}{2\mu + \lambda - g^2|\mathbf{q}|/v_F}}$$



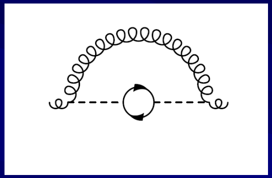


ELASTIC PROPERTIES OF GRAPHENE

For computational purposes, it is convenient to expand the coupling function $K(\mathbf{q})$ into a series of couplings for increasingly irrelevant operators

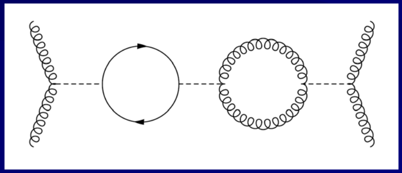
$$K(\mathbf{q}) = 2\mu + \lambda - \frac{g^2}{v_F}|\mathbf{q}| - \frac{(\lambda - g^2|\mathbf{q}|/v_F)^2}{2\mu + \lambda - g^2|\mathbf{q}|/v_F} \equiv \sum_n K_n |\mathbf{q}|^n$$

The irrelevant character of additional couplings is manifest in the self-energy corrections:



$$\sim \mathbf{p}^4 \frac{K_n}{\rho} \int d\omega\, d^2q \frac{|\mathbf{q}|^n}{\omega^2 - \varepsilon^2(\mathbf{q})} \sim \mathbf{p}^4 \frac{K_n}{\rho} \int d^2q \frac{|\mathbf{q}|^n}{\varepsilon(\mathbf{q})} \sim \mathbf{p}^4 \frac{K_n}{\sqrt{\rho\kappa}} \int^{q_c} dq\, |\mathbf{q}|^{n-1}$$

The couplings are themselves renormalized according to the corrections



$$\sim \frac{1}{\sqrt{\rho}\kappa^{3/2}} \sum_{i+j=n} K_i K_j \int^{q_c} dq \frac{1}{|\mathbf{q}|}$$

We can write down the hierarchy of scaling equations

$$q\frac{\partial\kappa}{\partial q} = -\frac{3}{16\pi} \sum_n q^n \frac{K_n}{\sqrt{\rho\kappa}}$$

$$q\frac{\partial K_n}{\partial q} = \frac{3}{64\pi} \frac{1}{\sqrt{\rho}\kappa^{3/2}} \sum_{i+j=n} K_i K_j$$

ELASTIC PROPERTIES OF GRAPHENE

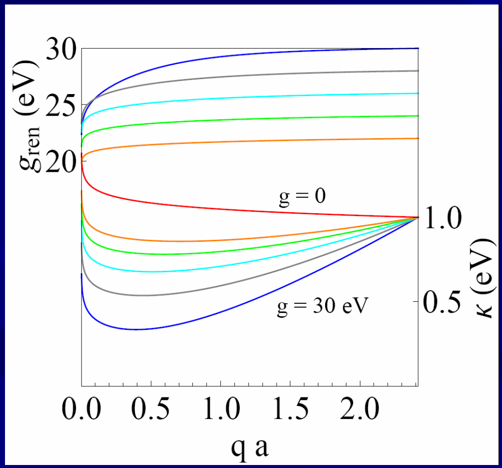
In practice, we have dealt with an expansion of the coupling function $K(\mathbf{q})$ to second order

$$K(\mathbf{q}) = K_0 + K_1|\mathbf{q}| + K_2|\mathbf{q}|^2 + O\left(|\mathbf{q}|^3\right)$$

with the respective couplings given in terms of the electron-phonon coupling g by

$$K_0 = 4\mu\frac{\mu+\lambda}{2\mu+\lambda}\ (\approx 40.3\,\mathrm{eV/A}^2)\ ,\ K_1 = -\frac{4\mu^2}{(2\mu+\lambda)^2}\frac{g^2}{v_F}\ (\approx 0.16 g^2\,\mathrm{eV/A})\ ,\ K_2 = -\frac{4\mu^2}{(2\mu+\lambda)^3}\frac{g^4}{v_F^2}\ (\approx 0.0009 g^4\,\mathrm{eV})$$

The resolution of the scaling equations gives rise to different regimes depending on g :



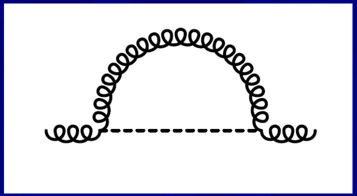
- at sufficiently large g, but still in the perturbative regime, we find a significant reduction of the bending rigidity
- we cannot access however the most interesting strong-coupling regime with $K_i/\rho^{1/2}\kappa^{3/2} \sim 1$, where it is likely a more drastic softening of the dispersion

(P. San-José, J. G. and F. Guinea, PRL 106, 045502 (2011))

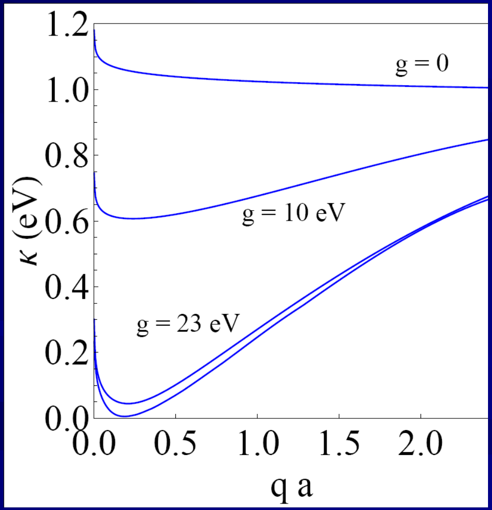
ELASTIC PROPERTIES OF GRAPHENE

In order to capture the physics of the strong-coupling regime, we can adopt an alternative nonperturbative approach based on a self-consistent solution of κ :

$$D^{-1}(\mathbf{q},\omega) = \rho\omega^2 - \kappa\,\mathbf{q}^4$$
$$\approx \rho\omega^2 - \kappa_0\mathbf{q}^4 - \mathbf{q}^4\frac{1}{2}\int\frac{d^2p}{(2\pi)^2}\frac{\sin^4(\theta)\,\mathbf{p}^2}{(\mathbf{p}-\mathbf{q})^4}\frac{K_0 + K_1|\mathbf{p}-\mathbf{q}| + K_2|\mathbf{p}-\mathbf{q}|^2}{\sqrt{\rho\kappa}}$$



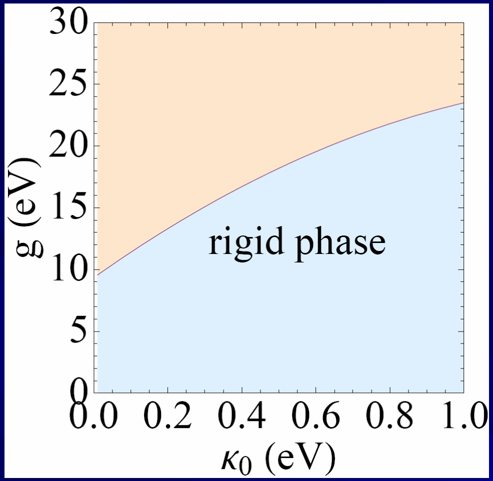
The resolution gives rise to a momentum-dependent bending rigidity $\kappa(\mathbf{q})$:



- for $g = 0$, we find a hardening of the dispersion extending the analysis of Nelson and Peliti at $T = 0$
- at moderate deformation potential $g > 10$ eV, there is an intermediate length scale where the rigidity is sensibly reduced
- at some critical g , $\kappa(\mathbf{q})$ is driven to zero, marking the point beyond which a real self-consistent solution does not exist

(P. San-José, J. G. and F. Guinea, PRL 106, 045502 (2011))

ELASTIC PROPERTIES OF GRAPHENE

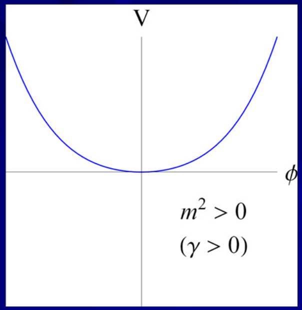


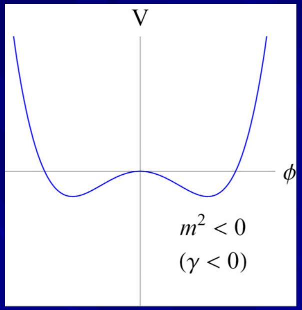
We can therefore characterize a rigid phase of graphene where it exists as a flat membrane over long-distance scales.

However, at the phase boundary we still have to make sense of a membrane with vanishing rigidity (and divergent fluctuations of the out-of-plane phonon field)

SPONTANEOUS SYMMETRY BREAKING

The model of flexural phonons becomes singular much in the same way as that of a scalar field with zero or negative mass square, in which the theory quantized around the trivial vacuum $\phi = 0$ is plagued by infrared divergences and interactions play an important role in stabilizing the effective potential.





There is actually a correspondence between the interacting model of flexural phonons and the relativistic scalar field theory with cuartic interaction

$$S = \frac{1}{2}\int dt\, d^2x \left(\rho(\partial_t h(\mathbf{x}))^2 - \kappa(\partial\,\partial h(\mathbf{x}))^2 - \gamma(P\,\partial h(\mathbf{x})\,\partial h(\mathbf{x})) - \frac{1}{8} K_0 (P\,\partial h(\mathbf{x})\,\partial h(\mathbf{x}))^2 \right)$$

$$S = \frac{1}{2}\int dt\, d^Dx \left((\partial_t \phi(\mathbf{x}))^2 - (\partial\phi(\mathbf{x}))^2 - m^2(\phi(\mathbf{x}))^2 - \lambda\,(\phi(\mathbf{x}))^4 \right)$$

that, for $D = 3$, have also in common their renormalizability at the quantum level.

SPONTANEOUS SYMMETRY BREAKING

It can be shown that the effective action

$$S_{\text{eff}}\left[h_{av}(\mathbf{x})\right]=\sum_{N=1}^{\infty}\frac{1}{N!}\sum_{i_1\ldots i_N}\int d\mathbf{x}_1\ldots d\mathbf{x}_N\,\Gamma^{(N)}_{i_1\ldots i_N}(\mathbf{x}_1,\ldots,\mathbf{x}_N)\,h_{av}(\mathbf{x}_1)\ldots h_{av}(\mathbf{x}_N)$$

has actually the same structure than that of the relativistic scalar theory (at least to leading order in a $1/d$ expansion for large number of field components).

The effective action $S_{\text{eff}}[h_{\text{av}}]$ satisfies in general

$$\frac{\delta S_{\text{eff}}\left[h_{av}(\mathbf{x})\right]}{\delta h_{av}}=0 \qquad , \qquad h_{av}(\mathbf{x})=\langle h(\mathbf{x})\rangle$$

so that minima with $h_{\text{av}}\neq 0$ are the signal of spontaneous symmetry breaking.

In our case it proves useful to rewrite the interaction with an auxiliary field σ

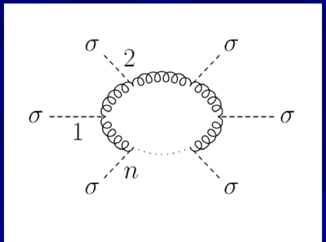
$$\int [d\,h(\mathbf{x})]\,e^{-\frac{1}{2}\int dt\,d^2x\,K_0\left(\frac{1}{2}P^T_{ij}\partial_i h(\mathbf{x})\partial_j h(\mathbf{x})\right)^2}=\int [d\,h(\mathbf{x})][d\,\sigma(\mathbf{x})]\,e^{\frac{1}{2}\int dt\,d^2x\left((\sigma(\mathbf{x}))^2-\sqrt{K_0}\,\sigma(\mathbf{x})\left(\frac{1}{2}P^T_{ij}\partial_i h(\mathbf{x})\partial_j h(\mathbf{x})\right)\right)}$$

The effective action is obtained by shifting by the average field $h(\mathbf{x})=h_{\text{av}}(\mathbf{x})+h_{\text{q}}(\mathbf{x})$, and it can be computed exactly in the limit of large d by integration of the quantum $h_{\text{q}}(\mathbf{x})$ fields.

SPONTANEOUS SYMMETRY BREAKING

Upon integration of the $h_q(\mathbf{x})$ field, we get:

$$V_{\text{eff}}[h_{av},\sigma_0] = \frac{1}{8\pi^2}\left(-\sigma_0{}^2 + 2\sqrt{K_0}\, u_{\text{av},|\mathbf{q}|<\Delta}\,\sigma_0\right) - i\sum_{n=2}^{\infty}\sigma_0{}^n \frac{1}{n}\frac{K_0^{n/2}}{(2\pi)^n}\int_{|\mathbf{p}|>\Delta}\frac{d^2p}{(2\pi)^2}\frac{d\omega}{2\pi}\frac{\mathbf{p}^{2n}}{(\rho\omega^2-\kappa\mathbf{p}^4)^n}$$



$$\sigma_0 \equiv \Delta^2\,\sigma_{|\mathbf{q}|<\Delta} \qquad , \qquad u_{\text{av}}(\mathbf{x}) = \Delta^2 P\,\partial h_{\text{av}}(\mathbf{x})\,\partial h_{\text{av}}(\mathbf{x})$$

The perturbative series has to be summed first to avoid the infrared divergences (P. San-José, J. G. and F. Guinea, PRL 106, 045502 (2011)):

$$V_{\text{eff}}[h_{av},\sigma_0] = \frac{1}{8\pi^2}\left(-\sigma_0{}^2 + 2\sqrt{K_0}\, u_{\text{av},|\mathbf{q}|<\Delta}\,\sigma_0\right) + \int\frac{d^2p}{(2\pi)^2}\frac{d\omega}{2\pi}\left(-\frac{\sqrt{K_0}}{2\pi}\sigma_0\frac{\mathbf{p}^2}{\rho\omega^2+\kappa\mathbf{p}^4} + \log\left(1+\frac{\sqrt{K_0}}{2\pi}\sigma_0\frac{\mathbf{p}^2}{\rho\omega^2+\kappa\mathbf{p}^4}\right)\right)$$

$$= \frac{1}{8\pi^2}\left(-\sigma_0{}^2 + 2\sqrt{K_0}\, u_{\text{av},|\mathbf{q}|<\Delta}\,\sigma_0\right) + \frac{1}{8\pi^2}\frac{K_0}{16\pi\sqrt{\rho}\kappa^{3/2}}\sigma_0{}^2\log\left(\frac{\sqrt{K_0}}{8\pi\sqrt{\rho\kappa}}\frac{\sigma_0}{\Lambda_c}\right)$$

This logarithmic dependence coincides precisely with the expression of the effective potential of a relativistic scalar theory in 3+1 dimensions (S. Coleman, R. Jackiw, H. Politzer, PRD 10, 2491 (1974)).

We can borrow then the knowledge about Higgs condensation in relativistic scalar field theories, to describe now the development of a nonvanishing expectation value $P\partial h_{\text{av}}\partial h_{\text{av}} \neq 0$.

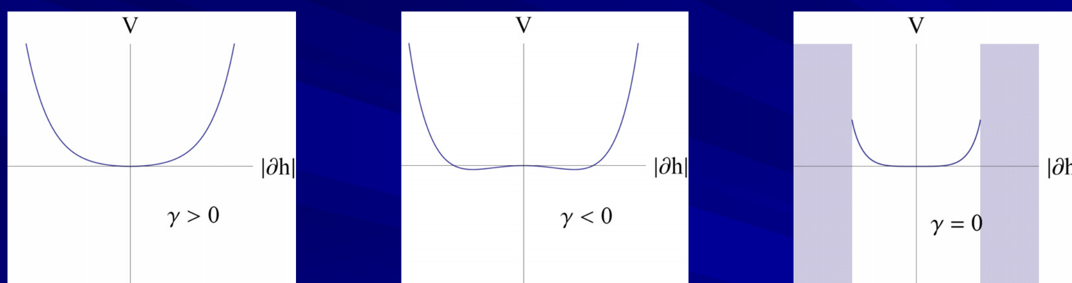
SPONTANEOUS SYMMETRY BREAKING

The tension γ plays here the same role as the mass square m^2 of the Higgs, and it drives from the phase without symmetry breaking to the phase with scalar field condensation:

$$V_{\text{eff}}\left[h_{av},\sigma_0\right]=\frac{1}{8\pi^2}\left(-\sigma_0{}^2+2\sqrt{K_0}\,u_{\text{av},|\mathbf{q}|<\Delta}\,\sigma_0+\gamma\,u_{\text{av},|\mathbf{q}|<\Delta}\right)+\frac{1}{8\pi^2}\frac{K_0}{16\pi\sqrt{\rho\kappa}^{3/2}}\sigma_0{}^2\log\left(\frac{\sqrt{K_0}}{8\pi\sqrt{\rho\kappa}}\frac{\sigma_0}{\Lambda_c}\right)$$

$$u_{\text{av}}(\mathbf{x})=\Delta^2 P\,\partial h_{av}(\mathbf{x})\,\partial h_{av}(\mathbf{x})$$

After integrating the auxiliary field σ we get the effective potential as a function of $P\partial h_{\text{av}}\partial h_{\text{av}}$:



For $m^2=0$ ($\gamma=0$ in our case), it is still controversial whether spontaneous symmetry breaking may take place in the Higgs model, as V_{eff} becomes complex above some value of the average field, pointing at an instability of the theory (at least in the $1/N$ approximation).

In conclusion:

- For physically sensible values of the graphene electron-phonon coupling g ($\gtrsim$ 23 eV), the system of flexural phonons shows a singularity (zero) in the bending rigidity κ .

- Resummation of perturbation theory shows that the singularity can be integrated out, but at the expense of introducing tension in the model, which leads to the possibility of describing spontaneous symmetry breaking for $\gamma < 0$.

- The discussion of whether condensation takes place for $\gamma = 0$ bring us back to the similar question in the massless Higgs theory, allowing to pose it as an experimentally addressable problem in graphene.

- Overall, we have a consistent mechanism for the development of ripples in graphene as a phenomenon of condensation of vortices for the gradient of the flexural phonon field, which should persist at temperature $T \neq 0$.