

FLAT BAND OF MIDGAP ROTATING SURFACE STATES IN 3D DIRAC AND WEYL SEMIMETALS UNDER RADIATION



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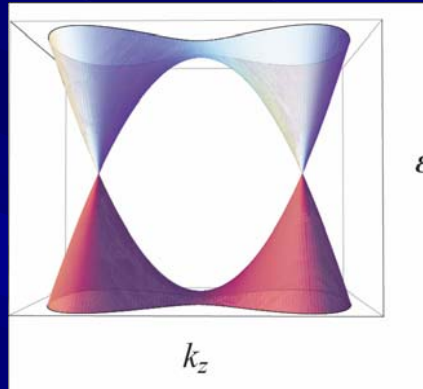
3D SEMIMETALS

We start with a model of 3D Dirac semimetal

$$H_0 = M(\mathbf{k}) \sigma_z + \hbar v (\zeta k_x \sigma_x + k_y \sigma_y) \quad \zeta = \pm 1$$

$$M(\mathbf{k}) = m_0 - m_1 k_z^2 - m_2 (k_x^2 + k_y^2)$$

The low-energy spectrum has two Dirac points at $\mathbf{k}_c = (0, 0, \pm \sqrt{m_0/m_1})$



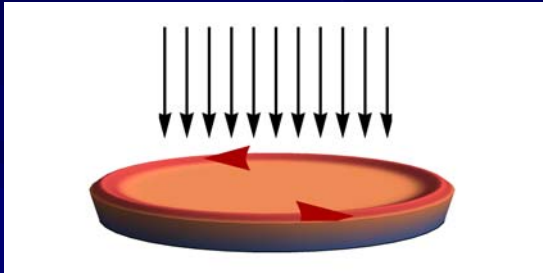
Near the Dirac points, the dispersion becomes linear in all directions in momentum space

$$\varepsilon_0(\mathbf{k}) = \pm \sqrt{\hbar^2 v^2 (k_x^2 + k_y^2) + 4m_0 m_1 (k_z - k_c)^2}$$

■ Z. Wang *et al.*, Phys. Rev. B 85, 195320 (2012).

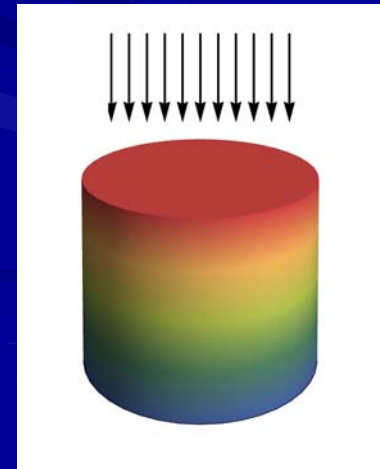
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2D AND 3D SEMIMETALS UNDER CIRCULARLY POLARIZED RADIATION



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3D DIRAC SEMIMETALS UNDER RADIATION

We investigate the changes in the spectrum when illuminating with circularly polarized light, corresponding to a vector potential

$$\mathbf{A}(t) = A(\eta \sin(\Omega t), \cos(\Omega t), 0) \quad \eta = \pm 1$$

We use Floquet theory to find the solutions of the time-dependent Schrödinger equation

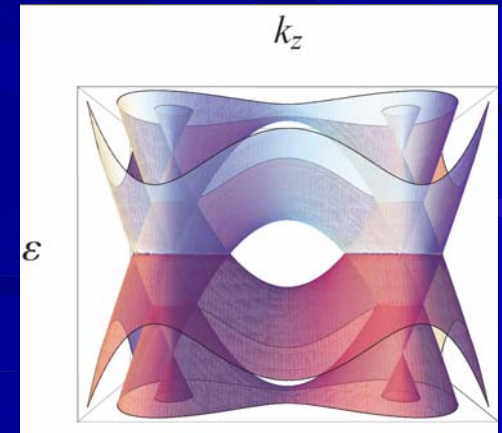
$$\begin{aligned} -i\hbar\partial_t |\Psi(t)\rangle &= H(t) |\Psi(t)\rangle \\ |\Psi(t)\rangle &= e^{-i\varepsilon t/\hbar} |\Phi(t)\rangle, \quad |\Phi(t+T)\rangle = |\Phi(t)\rangle \quad T = 2\pi/\Omega \end{aligned}$$

which amounts to find the result of hybridizing a number of Floquet subbands

$$|\Phi(t)\rangle = \sum_m e^{-im\Omega t} |\chi_m^{(\alpha)}\rangle,$$

$$\sum_n H_{mn} |\chi_n^{(\alpha)}\rangle = (\varepsilon_\alpha + m\hbar\Omega) |\chi_m^{(\alpha)}\rangle$$

$$H_{mn} = (1/T) \int_0^T dt H(t) e^{i(m-n)\Omega t}$$



3D DIRAC SEMIMETALS UNDER RADIATION

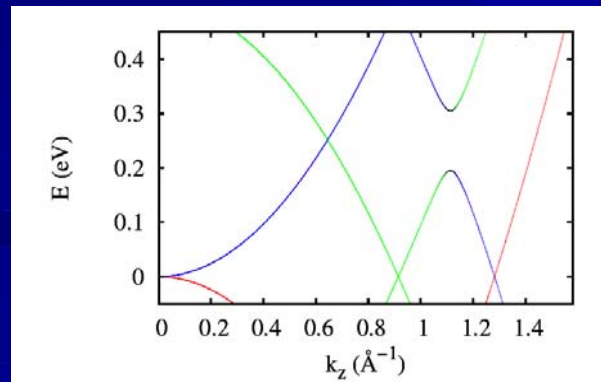
$$\sum_n H_{mn} \left| \chi_n^{(\alpha)} \right\rangle = (\varepsilon_\alpha + m\hbar\Omega) \left| \chi_m^{(\alpha)} \right\rangle$$

In our model we have

$$\begin{aligned} H_{nn} &= H_0 + n\hbar\Omega \mathbf{1} - m_2 A^2 \sigma_z \\ H_{n+1,n} &= (\hbar v / 2) A (i\sigma_x - \sigma_y) + m_2 A (-ik_x - k_y) \sigma_z \\ H_{n-1,n} &= (\hbar v / 2) A (-i\sigma_x - \sigma_y) + m_2 A (ik_x - k_y) \sigma_z \end{aligned}$$

The hybridization of the Floquet subbands gives rise to a gap over an extended ring in momentum space:

- proportional to A between subbands $n = 0$ and $n = \pm 1$
- proportional to A^2 between subbands $n = -1$ and $n = 1$



$$\hbar v = 1.1 \text{ eV } \text{\AA}$$

$$\hbar\Omega = 0.5 \text{ eV}$$

$$A = 0.05 \text{ \AA}^{-1}$$

$$k_x = 0, k_y = 0$$

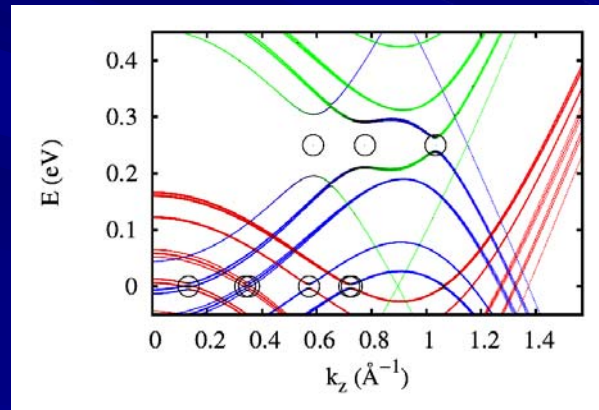
3D DIRAC SEMIMETALS UNDER RADIATION

But most interestingly, we can have evanescent states localized at the irradiated surface.

These states come as solutions with real quasi-energy ε_α for complex momenta k_z

$$\chi^{(\alpha)} \sim e^{ik_z z}, \quad k_z = k_R + ik_I$$

Computing for a wire with finite section in the x - y plane ($150 \times 150 \text{ \AA}^2$)



$$\begin{aligned} \hbar v &= 1.1 \text{ eV \AA} \\ \hbar \Omega &= 0.5 \text{ eV} \\ A &= 0.05 \text{ \AA}^{-1} \end{aligned}$$

we find evanescent states (for $\text{Im}(k_z) \approx 0.01 \text{ \AA}^{-1}$) at the position of the dots within the gap between Floquet subbands in the figure.

WEYL SEMIMETALS UNDER RADIATION

We can have a deeper insight by solving for a single Weyl quasiparticle

$$H_W = -i\hbar(v\sigma_x\partial_x + v\sigma_y\partial_y + v_z\sigma_z\partial_z) + \hbar v A(\sigma_x \cos(\Omega t) + \sigma_y \sin(\Omega t))$$

H_W can be translated to time-independent form by making a unitary transformation

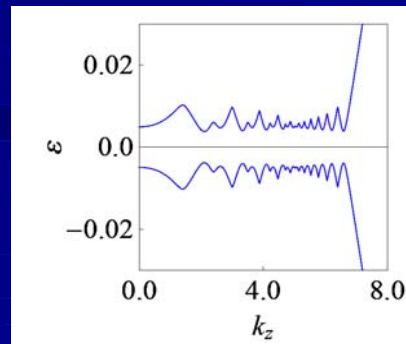
$$\begin{aligned}\tilde{H}_W &= U^\dagger H_W U - i\hbar U^\dagger \partial_t U & U &= e^{-iJ_z \Omega t / \hbar} \\ &= -i\hbar(v\sigma_x\partial_x + v\sigma_y\partial_y + v_z\sigma_z\partial_z) - \Omega J_z + \hbar v A \sigma_x\end{aligned}$$

and transforming again we obtain

$$\begin{aligned}\tilde{H}'_W &= P^\dagger \tilde{H}_W P & P &= e^{-iAx} \\ &= -i\hbar v \sum e^{\mp i\theta} (\partial_r \mp (i/r)\partial_\theta) \sigma_\pm - i\hbar v_z \sigma_z \partial_z \\ &\quad - \Omega J_z - \hbar \Omega A r \sin(\theta)\end{aligned}$$

$$J_z = -i\hbar \partial_\theta + \hbar \sigma_z / 2$$

We find that the spectrum has a gap between Floquet subbands with $j_z = \hbar/2$ and $j_z = -\hbar/2$



$$\hbar v = 1.1 \text{ eV } \text{\AA}$$

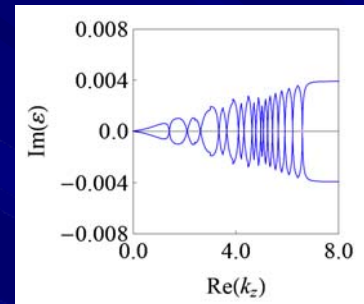
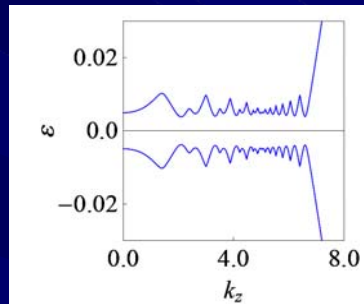
$$\hbar \Omega = 0.5 \text{ eV}$$

$$A = 0.005 \text{ } \text{\AA}^{-1}$$

WEYL SEMIMETALS UNDER RADIATION

The gap has oscillations as a function of k_z which are the signature of branch points in the complex plane

$$\varepsilon(k_z) \sim \sqrt{k_i - k_z} \sqrt{\bar{k}_i - k_z}$$



$$\hbar v = 1.1 \text{ eV } \text{\AA}$$

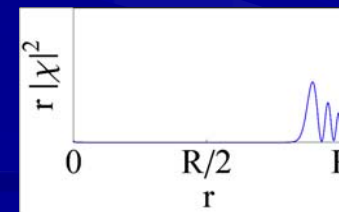
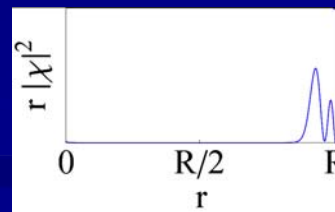
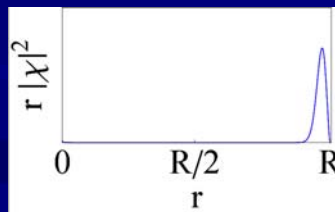
$$\hbar \Omega = 0.5 \text{ eV}$$

$$A = 0.005 \text{ \AA}^{-1}$$

$$\text{Im}(k_z) = 0.08 \text{ \AA}^{-1}$$

$k_i, \bar{k}_i$ are exceptional points at which $\varepsilon(k_z) = 0$, leading to a segment of real values of ε for $|\text{Im}(k_z)| \leq |\text{Im}(k_i)|$

The set of exceptional points corresponds to a sequence of evanescent states localized at the irradiated surface, with a peculiar quantization pattern



$$\text{Re}(k_1) > \text{Re}(k_2) > \text{Re}(k_3) \dots$$

WEYL SEMIMETALS UNDER RADIATION

The most important feature of the evanescent states is that they carry an angular current j^θ corresponding to the flow of probability density

$$\partial_t(\psi^\dagger \psi) \approx \partial_\theta j^\theta$$

j^θ can be computed in the space spanned by the states with $j_z = \pm \hbar/2$

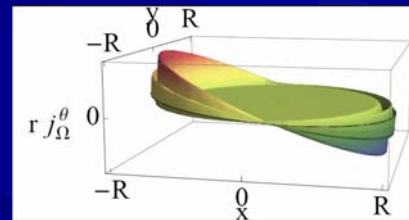
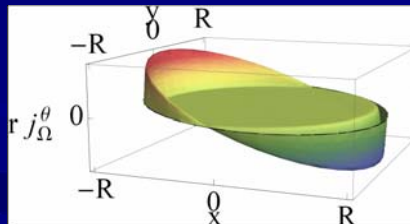
$$\chi = \begin{pmatrix} \phi_1(r) \\ e^{i\theta} \phi_2(r) \end{pmatrix} e^{ik_z z} + \begin{pmatrix} e^{-i\theta} \phi_3(r) \\ \phi_4(r) \end{pmatrix} e^{ik_z z}$$

The angular current has a uniform component, which is in general negligible,

$$j_{\text{static}}^\theta = -i \frac{v}{r} \left(\phi_1^*(r) \phi_2(r) + \phi_3^*(r) \phi_4(r) \right) + \text{h.c.}$$

and a time-dependent component

$$j_\Omega^\theta = -i \frac{v}{r} \left(\phi_1^*(r) \phi_4(r) e^{-i(\theta - \Omega t)} + \phi_3^*(r) \phi_2(r) e^{i(\theta - \Omega t)} \right) + \text{h.c.}$$



The corresponding current intensities have magnitudes $\int dr r j_\Omega^\theta \sim 10^{-3} v/a - 10^{-2} v/a$

3D SEMIMETALS UNDER CIRCULARLY POLARIZED RADIATION

In conclusion,

The irradiation of the surface of 3D semimetals must lead to the observation of evanescent states carrying a non negligible angular current. The feasibility of this observation may be supported by several facts:

- the limited screening in the 3D semimetals, which has to allow the infrared radiation to penetrate sufficiently deep ($\sim 1 \mu\text{m}$ at $\Omega \sim 100 \text{ THz}$)
- the magnitude of the angular current of individual states, which have associated a current intensity
$$e \int dr r j_{\Omega}^{\theta} \sim 10^{-1} \mu\text{A} - 1 \mu\text{A}$$
- the possibility of devising an experimental setup to work as a rectenna, benefiting from recent advances to rectify currents oscillating at the frequency of visible light

