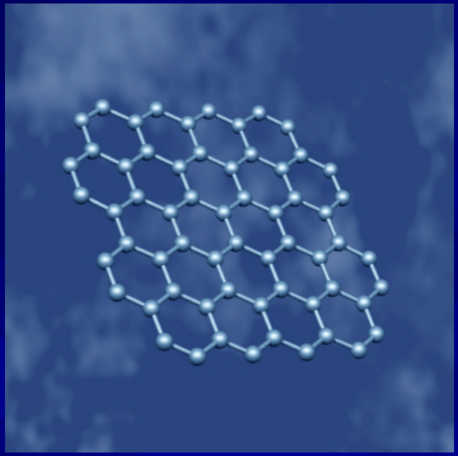
ELECTRONIC TRANSPORT IN GRAPHENE



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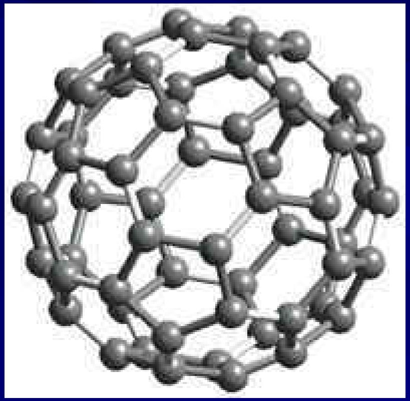
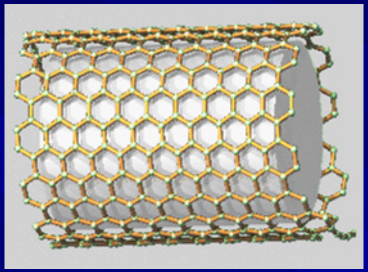
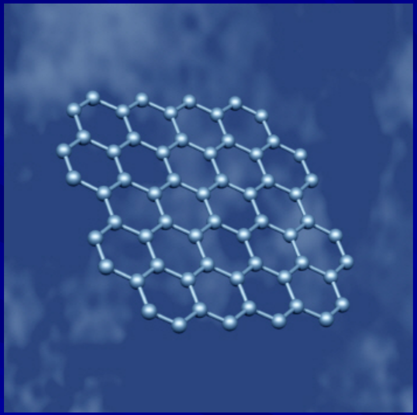






1985

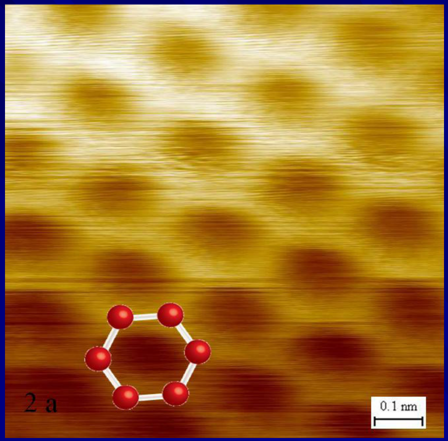
1991

2004

ELECTRONIC TRANSPORT IN GRAPHENE

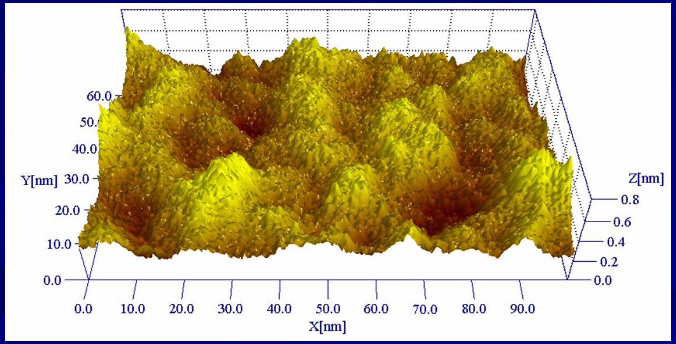
Graphene has opened the way to understand the behavior of an electron system in D = 2

- remarkable properties are observed from the theoretical point of view
- it has sparked great expectations of reaching very large mobilities



From E. Stolyarova *et al.*,

Proc. Natl. Acad. Sci. 104, 9209 (2007)



From E. Stolyarova *et al.*,

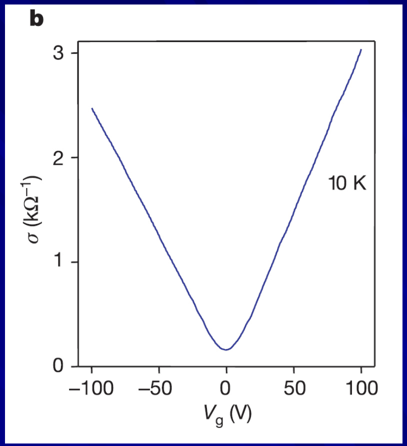
Proc. Natl. Acad. Sci. 104, 9209 (2007)

But … some challenges have to be faced:

- samples have significant corrugation
- the interaction with the substrate and boundary conditions modify significantly the transport properties

ELECTRONIC TRANSPORT IN GRAPHENE

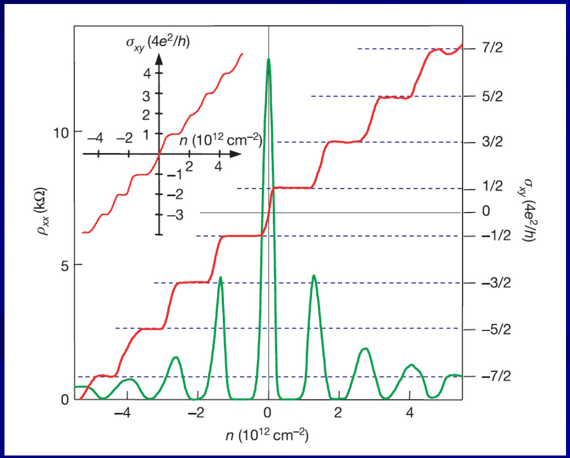
The first experimental observations and measurement of unusual transport properties pointed at the existence of a conical dispersion of quasiparticles in graphene



From K. S. Novoselov *et al.*, Nature 438, 197 (2005)

- the electric field effect shows that a substantial concentration of electrons (holes) can be induced by changes in the gate voltage

$$n \propto V_g$$



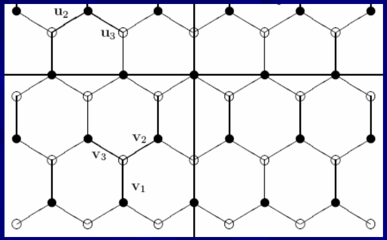
From K. S. Novoselov *et al.*, Nature 438, 197 (2005)

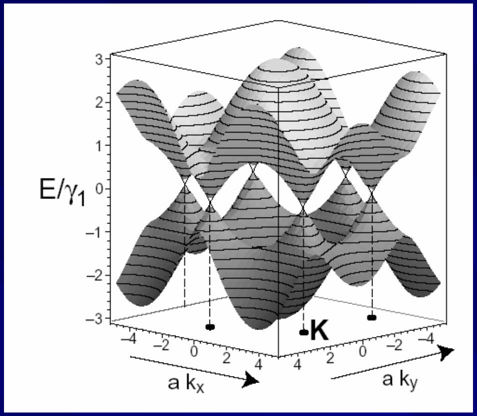
- the response to a magnetic field is also unusual, as observed in particular in the quantum Hall effect

$$\sigma_{xy} = \frac{4e^2}{h}(N + 1/2)$$

ELECTRONIC TRANSPORT IN GRAPHENE

The observed properties were actually consistent with the dispersion expected for electrons in a honeycomb lattice





- The undoped electron system has isolated Fermi points at the corners of an hexagonal Brillouin zone

- The conical dispersion is a genuine property of two-component fermions (Dirac fermions) with hamiltonian

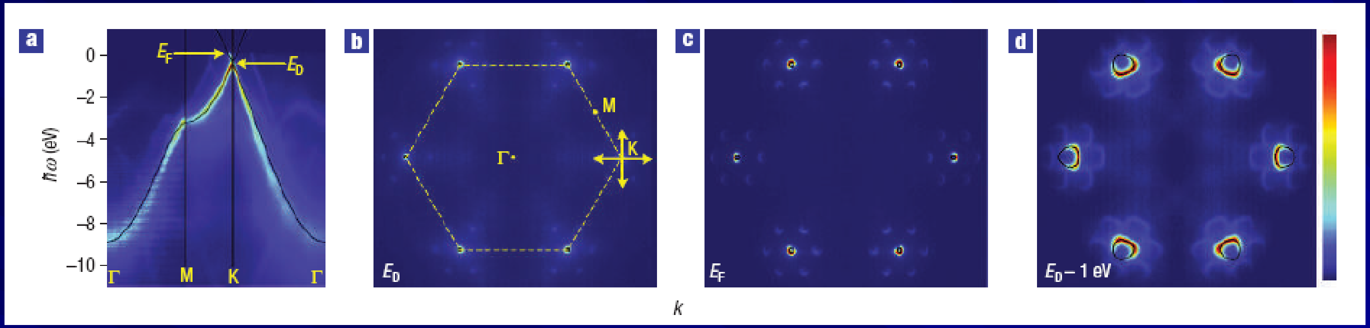
$$H = v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}$$

We have to introduce a Dirac fermion for each independent Fermi point, at which

$$\varepsilon(\mathbf{k}) = \pm v_F |\mathbf{k}| \qquad , \qquad n(\varepsilon) \propto |\varepsilon|$$

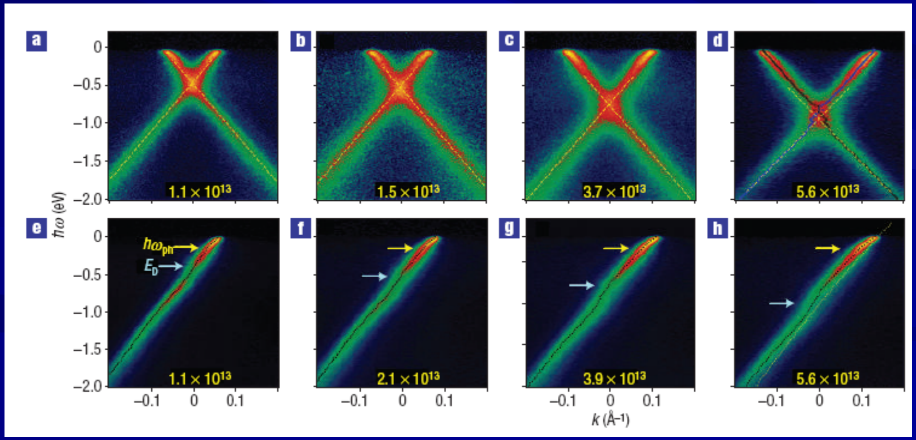
ELECTRONIC TRANSPORT IN GRAPHENE

A direct evidence of the conical dispersion has been obtained with angle-resolved photoemission spectroscopy



A. Bostwick *et al.*, Nature Phys. 3, 36 (2007)

These experiments are also useful to provide a measure of the interaction effects in graphene



A. Bostwick *et al.*, Nature Phys. 3, 36 (2007)

QUANTUM HALL EFFECT IN GRAPHENE

The peculiar quantization of the Hall conductivity can be explained satisfactorily by the coupling of the Dirac fermions to gauge fields:

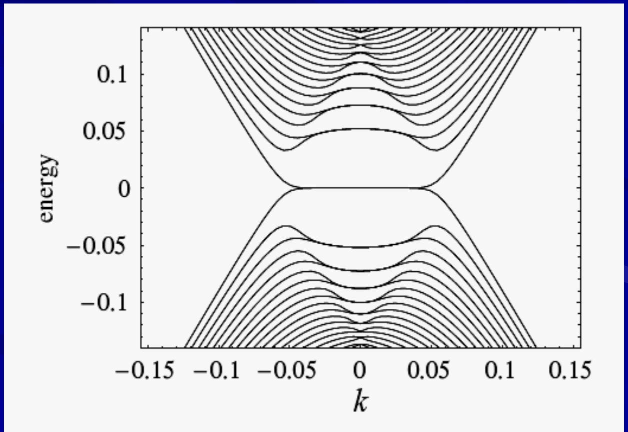
$$H_{tb} = -t \sum_{r,r'} \psi^{+}(\mathbf{r}') \; \exp(i(e/\hbar c)\int_{r}^{r'} \mathbf{A}\cdot d\mathbf{l}) \; \psi(\mathbf{r})$$

which corresponds to the gauge prescription

$$H = v_F \, \boldsymbol{\sigma}\cdot\mathbf{k} \quad \rightarrow \quad v_F \, \boldsymbol{\sigma}\cdot(\mathbf{k} - \tfrac{e}{c}\mathbf{A})$$

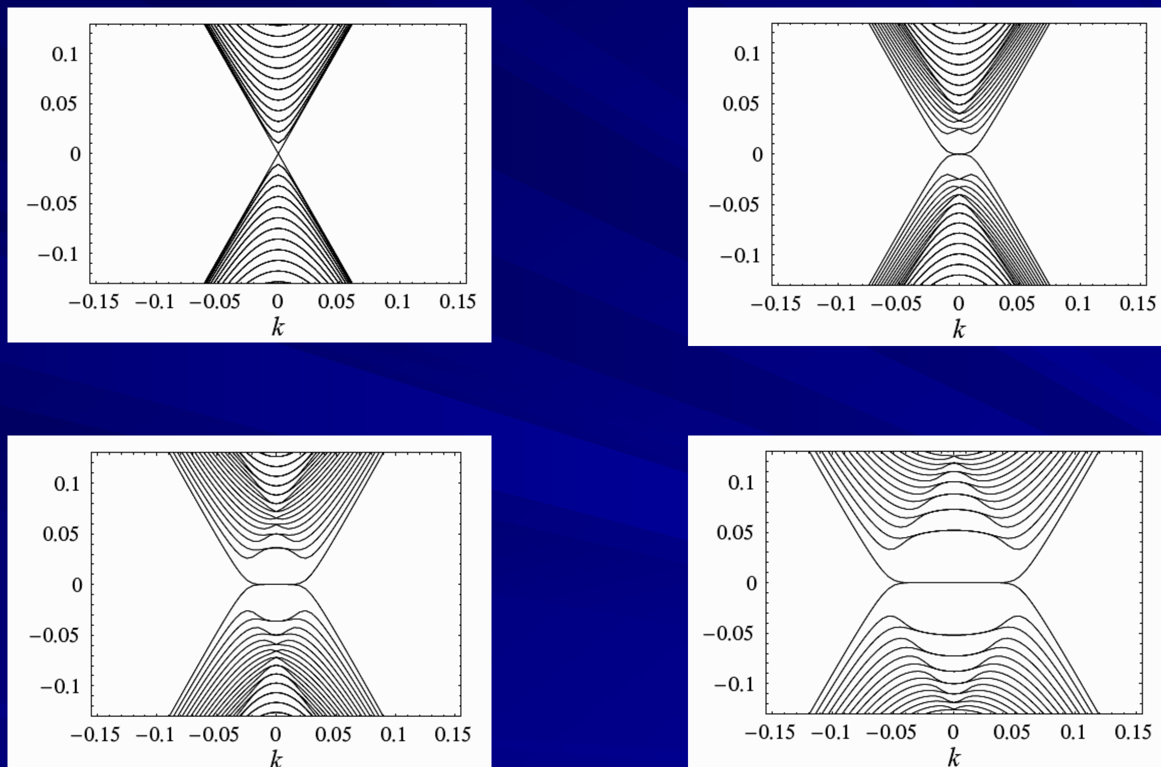
This leads to Landau levels quantized according to the expression

$$E_N = \mathrm{sgn}(N)\sqrt{2e\hbar v_F^2 |N| B}$$



QUANTUM HALL EFFECT IN GRAPHENE

The quantum Hall effect is actually quite robust and should persist even in the presence of curvature of the samples. In the case of the shells of MWNTs with radius $R = 20$ nm, for instance, we can predict the sequence of band structures for magnetic field strength $B = 0, 5, 10, 20$ T :



S. Bellucci, J. G., F. Guinea, P. Onorato and E. Perfetto,
J. Phys.: Condens. Matter **19**, 395017 (2007)

ELECTRONIC TRANSPORT IN GRAPHENE

However, there are open questions in the case of the conductivity. This is usually computed in Boltzmann transport theory in terms of the density of states *D* as

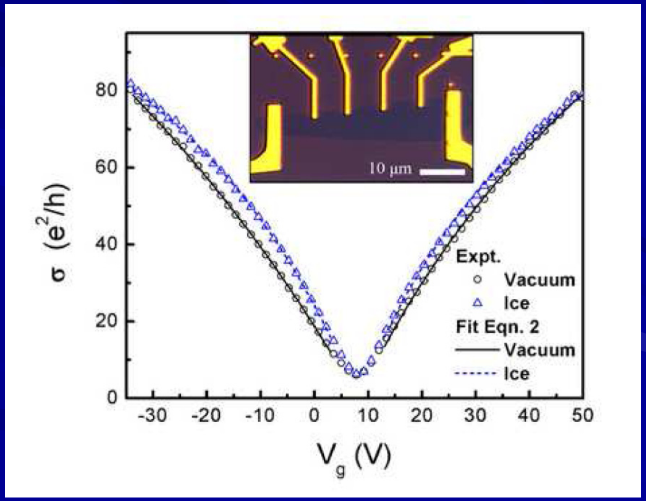
$$\sigma(\mu) = e^2\, D(\mu)\, v_F^2\, \tau(\mu)$$

The linear dependence observed experimentally on gate voltage implies that σ should be proportional to the electron density.

- in the case of short-range scatterers, $\tau \propto 1/k$, implies that $\sigma =$ const.
- in the case of charged long-range scatterers, $\tau \propto k$, implies that $\sigma \propto \mu^2$

This argument seems to be also in agreement with recent experiments where the influence of the background dielectric constant is shown. The best fit to the conductivity is given by

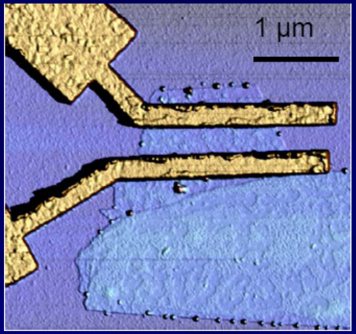
$$\frac{1}{\sigma(\mu)} = \frac{1}{e\, n(\mu)\, \mu_l} + \frac{1}{\sigma_s}$$

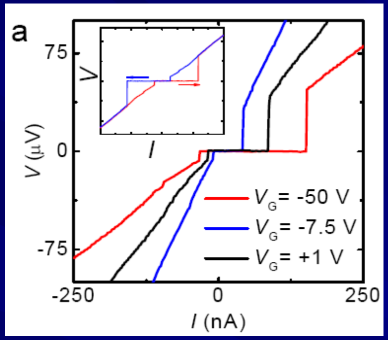


From C. Jang *et al.*, Phys. Rev. Lett. 101, 146805 (2008)

JOSEPHSON EFFECT IN GRAPHENE

There have been also observations of supercurrents, when graphene is contacted with superconducting electrodes

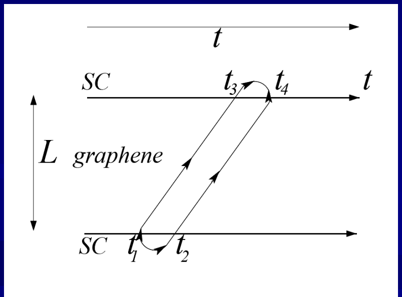


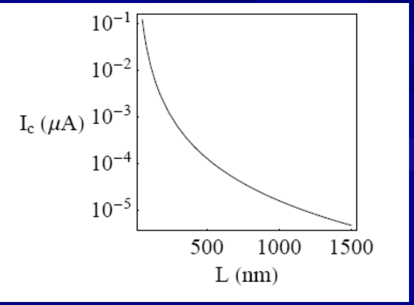


From H. B. Heersche *et al.*,

Nature **446**, 56 (2007)

The reason why supercurrents may exist at the Dirac point is that Cooper pairs have a nonvanishing propagation even at vanishing charge density



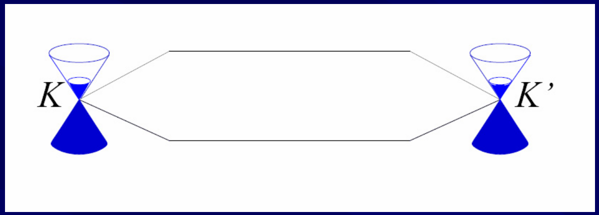


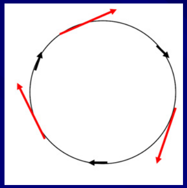
$$D^{(0)}(\mathbf{k},\omega)\Big|_{T=0} = -\frac{1}{8v_F^2}\sqrt{v_F^2\mathbf{k}^2-\omega^2}$$

From J. G. and E. Perfetto,
Phys. Rev. B 76, 155404 (2007)

ELECTRONIC TRANSPORT IN GRAPHENE

The scattering by impurities is quite unconventinal in graphene, due to the chirality of electrons. When a quasiparticle encircles a closed path in momentum space, it picks up a Berry phase of π

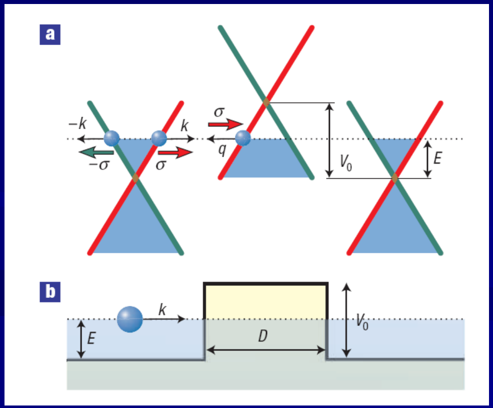


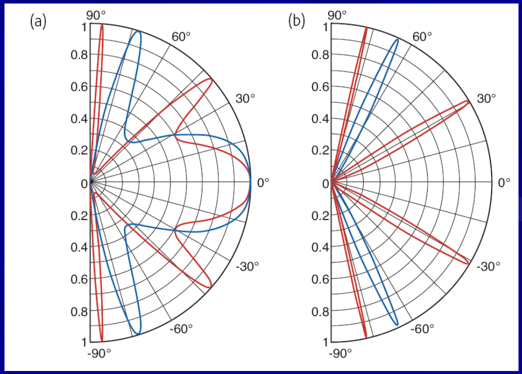


$$\Psi \rightarrow e^{i\pi}\ \Psi$$

In the absence of scatterers that may induce a large momentum-transfer, backscattering is then suppressed (H. Suzuura and T. Ando, Phys. Rev. Lett. 89,266603 (2002).

This also explains the peculiar properties of electrons when tunneling across potential barriers: the transmission probability is equal to 1 at normal incidence, and 0 for backscattering

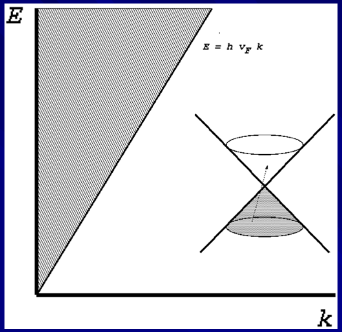




M. I. Katsnelson, K. S. Novoselov, and A. K. Geim, Nature Physics 2, 620 (2006)

MANY-BODY EFFECTS IN GRAPHENE

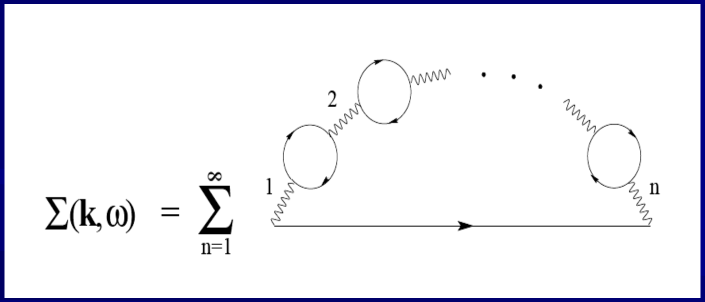
Graphene is a system with remarkable many-body properties, starting with the behavior of its electron-hole excitations. The polarization is



$$\Pi_0(\mathbf{q},\omega) = -\frac{\mathbf{q}^2}{8\sqrt{v_F^2\mathbf{q}^2 - \omega^2}}$$

In the undoped system, there are no electron-hole excitations nor plasmons into which the electrons can decay (J. G., F. Guinea and M.A.H. Vozmediano, Nucl . Phys. B424, 595 (1994))

The single-particle properties are significantly renormalized due to the strong Coulomb interaction:



$$\frac{1}{G} = \frac{1}{G_0} - \Sigma$$

$$\approx \omega_k - v_F \boldsymbol{\sigma}\cdot\mathbf{k}$$

$$-\,\omega_k\,\gamma(g)\log(E_c/\omega_k) - v_F\boldsymbol{\sigma}\cdot\mathbf{k}\,\beta(g)\log(E_c/\omega_k)$$

(J. G., F. Guinea and M. A. H. Vozmediano, Phys. Rev. B 59, 2474 (1999))

with $\quad g \equiv e^2/16 v_F$

MANY-BODY EFFECTS IN GRAPHENE

The imaginary part of the self-energy is $\mathrm{Im}\,\Sigma(\mathbf{k},\omega) \propto \left(\frac{e^2}{v_F}\right)^2 \omega \propto \frac{\omega}{\log^2(\omega)}$

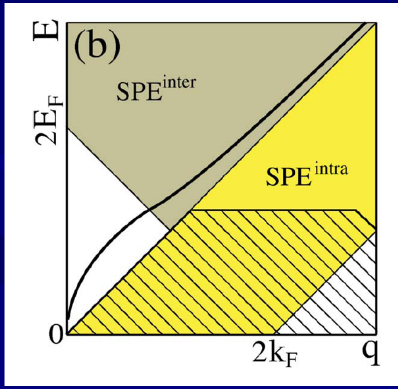
However, this does nor imply a linear QP decay, as reflected in the singular behavior

$$\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}| + \varepsilon) \propto v_F|\mathbf{k}|$$

$$\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}| - \varepsilon) = 0$$

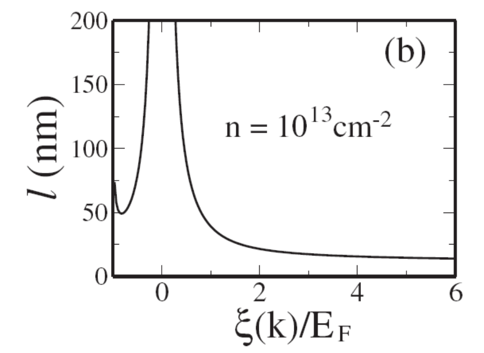
(J. G., F. Guinea and M. A. H. Vozmediano, Phys. Rev. Lett. **77**, 3589 (1996))

In the doped system, the decay of quasiparticles is possible due to intraband electron-hole excitations:



The QP decay rate is now:

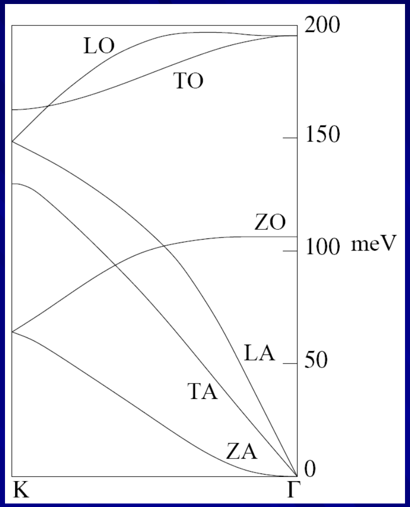
$$\tau^{-1} \propto (k - k_F)^2 \log(|k - k_F|)$$



E. H. Hwang, Ben Yu-Kuang Hu, and S. Das Sarma, Phys. Rev. B **76**, 115434 (2007)

MANY-BODY EFFECTS IN GRAPHENE

We now turn to phonons as the relevant source of scattering at low carrier densities. At sufficiently large energy/temperature we have the contribution of optical phonons. The QP decay rate is

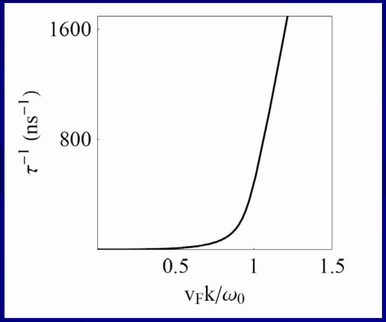


$$\tau^{-1} = -\,\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}|)$$
$$\propto \mathrm{Im}\, ig^2 \int d^2q \int d\omega_q \frac{\omega_k - \omega_q + v_F \boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})}{-(\omega_k - \omega_q)^2 + (\mathbf{k}-\mathbf{q})^2 - i\varepsilon} D(\mathbf{q}, \omega_q)$$

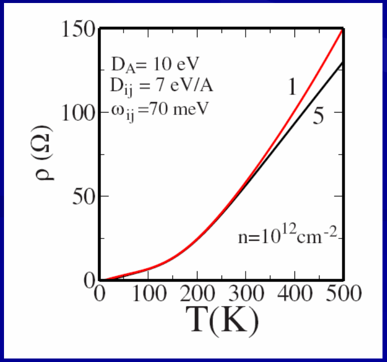
This gives rise to a decay rate linearly proportional to the QP energy, above the phonon energy.

Using Boltzmann transport theory, we obtain a resistivity that does not depend on carrier density and is lineraly proportional to temperature

$$\sigma(\mu) = e^2\, D(\mu)\, v_F^2\, \tau(\mu)$$



J. G., E. Perfetto, Phys. Rev. Lett. (in press)

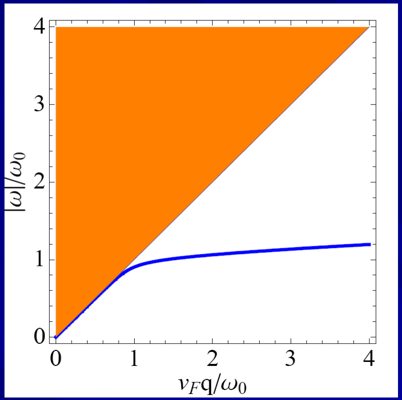


E. H. Hwang and S. Das Sarma, Phys. Rev. B **77**, 115449 (2008)

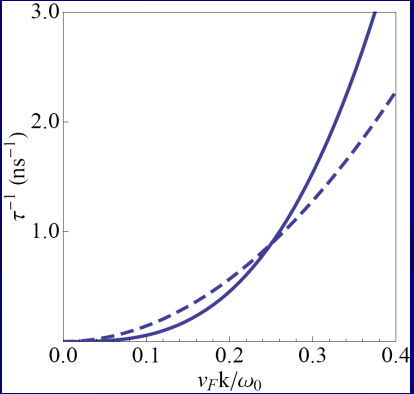
MANY-BODY EFFECTS IN GRAPHENE

There is also interesting physics below the scale of the out-of-plane phonons. These couple to the electron charge and have therefore a strong hybridization with electron-hole pairs. In the RPA,

$$D(\mathbf{q},\omega) \approx \frac{2\omega_0}{\omega^2 - \omega_0^2 + i\varepsilon - g^2 2\omega_0 \dfrac{\mathbf{q}^2/2 - \omega^2/v_F^2}{8\sqrt{v_F^2\mathbf{q}^2 - \omega^2}}}$$



We observe the appearance of very soft phonon modes near the K point of graphene, which are right below the particle-hole continuum

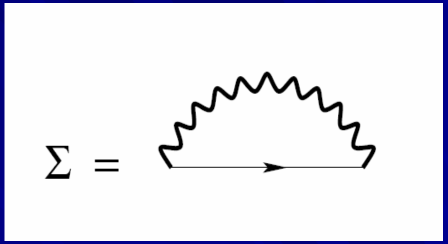


The hybrid states give rise to a cubic dependence of the QP decay rate on energy, that competes with the lower bound given at very low energies by the decay into acoustic phonons

J. G., E. Perfetto, Phys. Rev. Lett. (in press)

LOW-ENERGY ELECTRONIC PROPERTIES

The existence of soft phonon modes changes significantly the low-energy electronic properties. We have for instance the quasiparticle decay rate



$$\tau^{-1} = -\,\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}|)$$

$$\propto \mathrm{Im}\, i g^2 \int d^2 q \int d\omega_q \, \frac{\omega_k - \omega_q + v_F \boldsymbol{\gamma}^{(a)} \cdot (\mathbf{k}-\mathbf{q})}{-(\omega_k - \omega_q)^2 + (\mathbf{k}-\mathbf{q})^2 - i\varepsilon} D(\mathbf{q}, \omega_q)$$

$$\propto g^2 \int_0^{|\mathbf{k}|} dq \, |\mathbf{q}| \int_0^{|\mathbf{q}|} d\Omega_{\mathbf{q}} \left| \frac{\partial \phi}{\partial \Omega_{\mathbf{q}}} \right| \delta\big(Q(\mathbf{q}, \Omega_{\mathbf{q}})\big) \quad , \quad \text{where} \quad Q(\mathbf{q},\omega) = \frac{\omega^2 - \omega_0^2}{2\omega_0} - g^2 \frac{\mathbf{q}^2/2 - \omega^2/v_F^2}{8\sqrt{v_F^2 \mathbf{q}^2 - \omega^2}}$$

We have now quite different behaviors depending on the energy range:

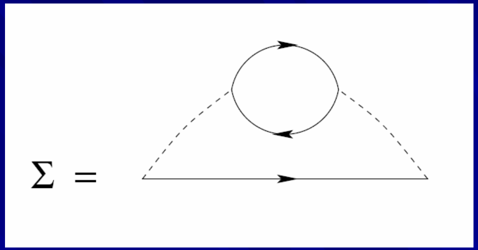
$$\tau^{-1} \propto \begin{cases} \left(\dfrac{g^2}{v_F^2}\right) v_F |\mathbf{k}| & v_F|\mathbf{k}| > \omega_0 \\[2ex] \left(\dfrac{g^2}{v_F^2}\right)^2 \dfrac{v_F^3 |\mathbf{k}|^3}{\omega_0^2} & v_F|\mathbf{k}| < \omega_0 \end{cases}$$

(consistent with C.-H. Park *et al.*, Phys. Rev. Lett. **99**, 086804 (2007))

J. G. and E. Perfetto, Phys. Rev. Lett. (in press)

LOW-ENERGY ELECTRONIC PROPERTIES

For comparison, we may derive the quasiparticle decay rate in the case of a screened Coulomb interaction:



$$\tau^{-1} = -\,\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}|)$$

$$\propto \mathrm{Im}\, ie^2 \int d^2q \int d\omega_q \frac{\omega_k - \omega_q + v_F \boldsymbol{\gamma}^{(a)} \cdot (\mathbf{k}-\mathbf{q})}{-(\omega_k - \omega_q)^2 + (\mathbf{k}-\mathbf{q})^2 - i\varepsilon} V(\mathbf{q}, \omega_q)$$

$$\propto e^4 \int_0^{|\mathbf{k}|} dq\, |\mathbf{q}| \int_{|\mathbf{q}|}^{|\mathbf{q}|+\delta} d\Omega_{\mathbf{q}} \left| \frac{\partial \phi}{\partial \Omega_{\mathbf{q}}} \right| \frac{1}{\mathbf{q}^2 + \ell^{-2}} \frac{\mathbf{q}^2}{\sqrt{\Omega_{\mathbf{q}}^2 - v_F^2 \mathbf{q}^2}}$$

In the limit $\delta \to 0$, we have a finite quasiparticle decay rate, with two different regimes

$$\tau^{-1} \propto \begin{cases} \left(\dfrac{e^2}{v_F}\right)^2 v_F |\mathbf{k}| & |\mathbf{k}| > \ell^{-1} \\[2ex] \left(\dfrac{e^2}{v_F}\right)^2 v_F \ell^2 |\mathbf{k}|^3 & |\mathbf{k}| < \ell^{-1} \end{cases}$$

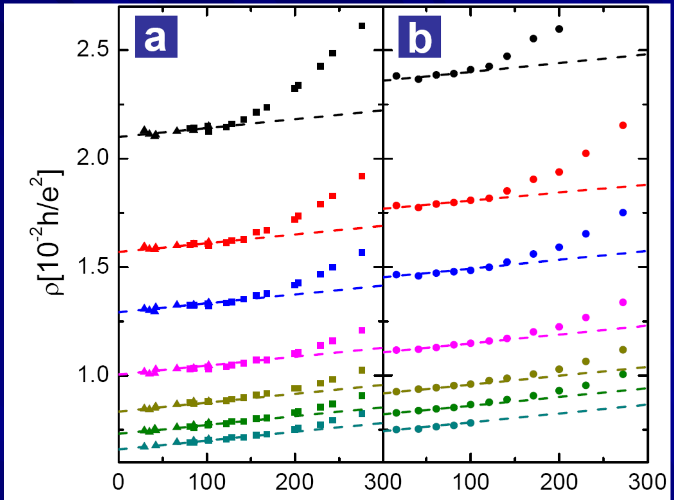
(J. G., F. Guinea and M. A. H. Vozmediano, Phys. Rev. Lett. **77**, 3589 (1996), also consistent with E. H. Hwang, B. Y.-K. Hu, and S. Das Sarma, Phys. Rev. B **76**, 115434 (2007))

RESISTIVITY AND MOBILITY IN GRAPHENE

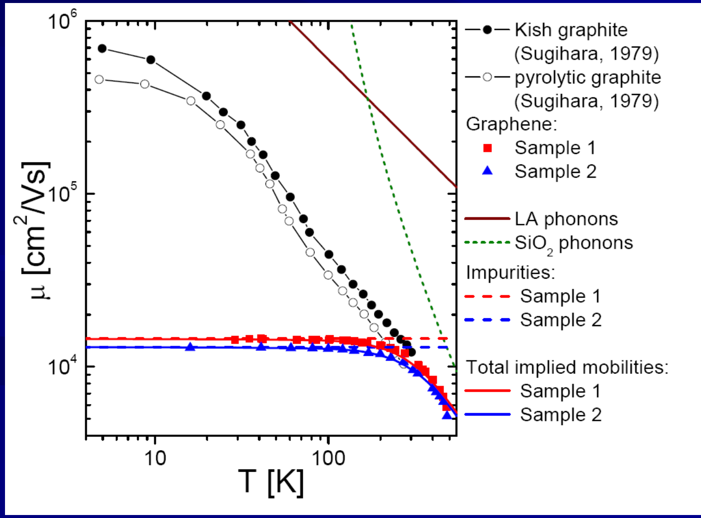
The theoretical results have to matched with the experimental measures of the resistivity.

It is assumed that the resistivity has a T-independent contribution from impurities, another from acoustic phonon scattering, and some extra contribution giving the nonlinear behavior

$$\rho(V_g, T) = \rho_{\mathrm{imp}}(V_g) + \rho_{\mathrm{ac}}(T) + \rho_{\mathrm{nl}}(V_g, T)$$



J.-H. Chen *et al.*, Nature Nanotech. 3, 206 (2008)



J.-H. Chen *et al.*, Nature Nanotech. 3, 206 (2008)

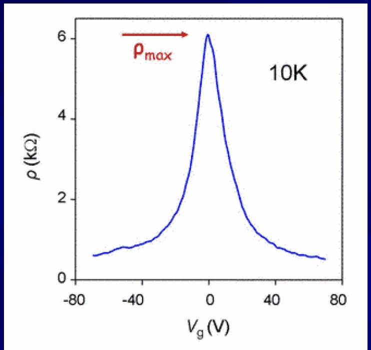
The hope is to be able to remove the contribution from impurities to remain with the intrinsic source of resistivity (phonons), in which case the mobility would diverge at low carrier density as

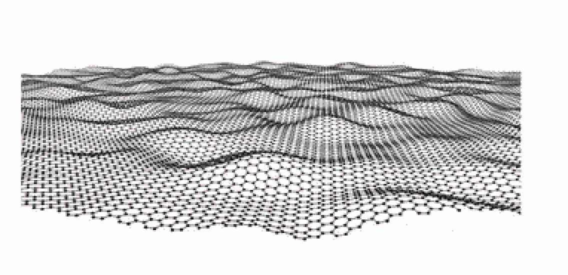
$$\mu = \frac{1}{ne\rho}$$

To conclude,

Graphene seems a quite exciting material from the experimental as well as from the theoretical point of view

- from the theoretical point of view, the question of the minimum conductivity is still a matter of debate





- from a practical point of view, one has to be able to tailor the graphene structure at the nanoscale, as well as to suppress the extrinsic scattering mechanisms that are the main source of resistivity at present