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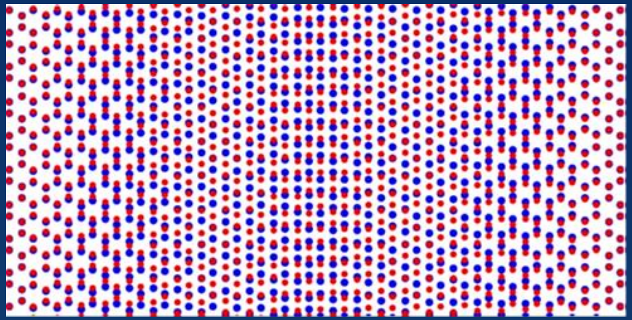
THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE



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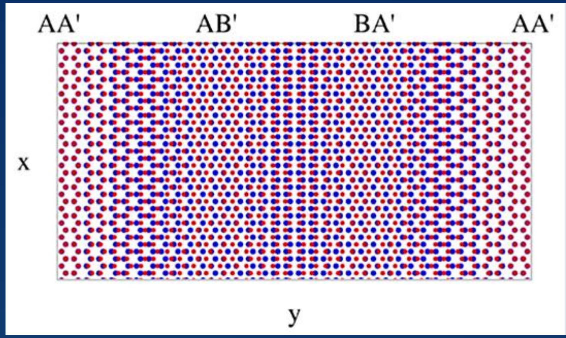
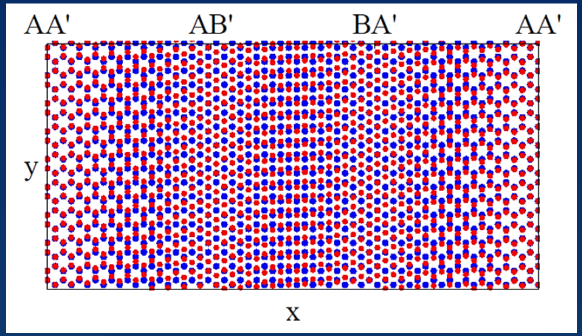
THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

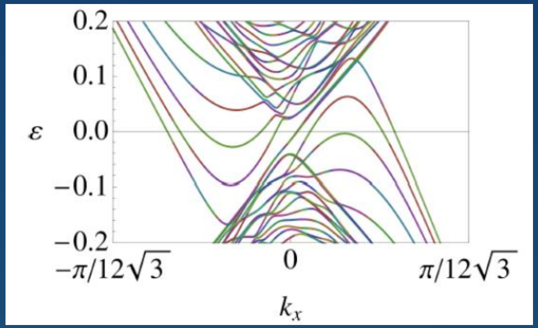
We investigate the effect of shear and the consequent moiré patterns which may arise in bilayer graphene, looking for similarities and deviations with respect to the phases in twisted bilayer graphene.

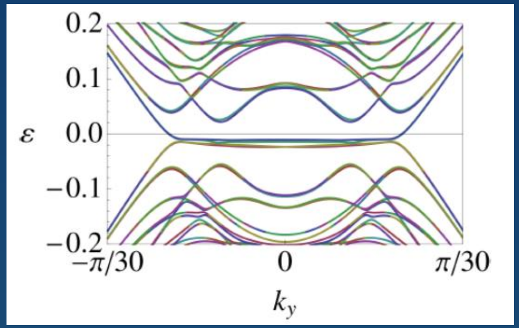
It is already known that strain or shear may produce 1D moiré patterns in bilayer graphene, with a sequence of *AA-AB-BA* stacking regions





However, the dispersion of the electron bands is very different in the cases of bilayer graphene with shear or strain

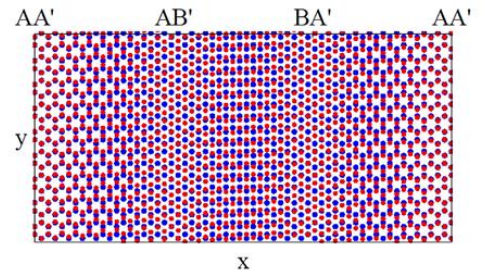




J. G., Phys. Rev. B 94, 165401 (2016)

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

The development of flat bands can be studied by means of a continuum approximation



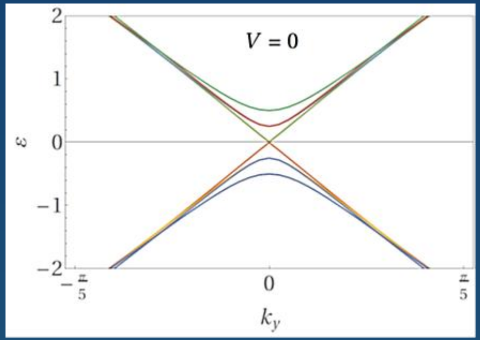
$$H = v_F \begin{pmatrix} 0 & -i\nabla_x^{(1)} - \nabla_y^{(1)} & V_{AA'}(\mathbf{r}) & V_{AB'}(\mathbf{r}) \\ -i\nabla_x^{(1)} + \nabla_y^{(1)} & 0 & V_{BA'}(\mathbf{r}) & V_{AA'}(\mathbf{r}) \\ V^{\star}_{AA'}(\mathbf{r}) & V^{\star}_{BA'}(\mathbf{r}) & 0 & -i\nabla_x^{(2)} - \nabla_y^{(2)} \\ V^{\star}_{AB'}(\mathbf{r}) & V^{\star}_{AA'}(\mathbf{r}) & -i\nabla_x^{(2)} + \nabla_y^{(2)} & 0 \end{pmatrix}$$

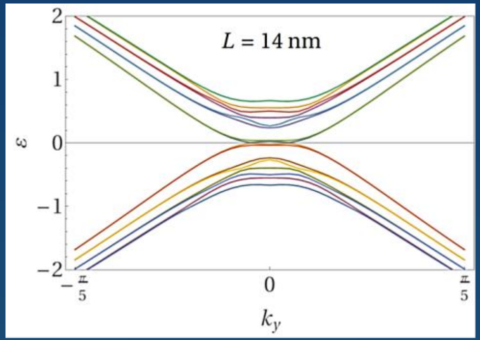
This model can be cast in terms of a fictitious non-Abelian gauge field $\hat{\mathbf{A}}$

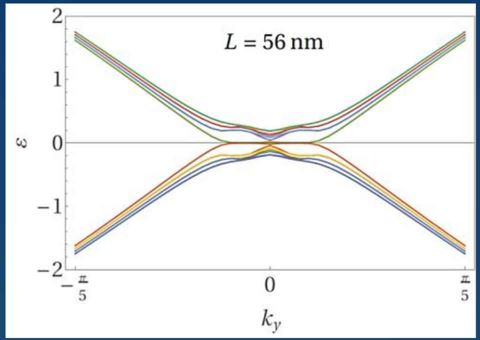
(P. San-José, J. G., F. Guinea, Phys. Rev. Lett. 108, 216802 (2012))

$$H = v_F \boldsymbol{\sigma} \cdot (-i\boldsymbol{\nabla} - \hat{\mathbf{A}}) + v_F V_{AA'} \tau_1$$

The fictitious gauge field is responsible for low-energy bands resembling those of the quantum Hall effect





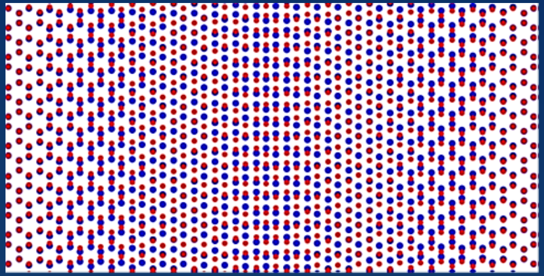


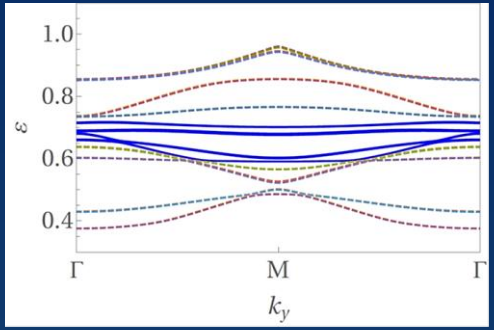
THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

In the tight-binding approximation, we can engineer a set of flat bands by applying a periodic longitudinal potential $\sin(2\pi y/L_y)$

$$H_0 = -\sum_{i,j} t(\mathbf{r}_i - \mathbf{r}_j)\left(a_{i\sigma}^{+} a_{j\sigma} + \text{h.c.}\right) + \sum_{i} \sin(2\pi y_i/L_y)\; a_{i\sigma}^{+} a_{i\sigma}$$

The new period in the y direction has the effect of folding the low-energy bands





J. G. and T. Stauber, arXiv:2503.05624

Next, we study interaction effects in the flat bands by including both long-range (screened) Coulomb and Hubbard contributions

$$H_{\text{int}} = H_C + H_U$$

$$H_C = \frac{1}{2} \sum_{i,j,\sigma,\sigma'} a_{i\sigma}^{+} a_{i\sigma}\, v_C(\mathbf{r}_i - \mathbf{r}_j)\, a_{j\sigma'}^{+} a_{j\sigma'} \qquad v_C(\mathbf{r}) = \frac{e^2}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + \xi^2}}\right)$$

$$H_U = U \sum_{i} a_{i\uparrow}^{+} a_{i\uparrow}\;\; a_{i\downarrow}^{+} a_{i\downarrow}$$

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

The noninteracting Hamiltonian H_0 can be written in terms of the eigenvalues and eigenvectors of the large tight-binding matrix

$$(H_0)_{i\sigma,j\sigma} = \sum_a \varepsilon^{(0)}_{a\sigma}\, \varphi^{(0)}_{a\sigma}(\mathbf{r}_i)\, \varphi^{(0)}_{a\sigma}(\mathbf{r}_j)^*$$

The noninteracting electron propagator G_0 becomes the inverse of H_0 in the zero-frequency (static) limit

$$(G_0)_{i\sigma,j\sigma} = -\sum_a \frac{1}{\varepsilon^{(0)}_{a\sigma}}\, \varphi^{(0)}_{a\sigma}(\mathbf{r}_i)\, \varphi^{(0)}_{a\sigma}(\mathbf{r}_j)^*$$

The Hartree-Fock approximation proceeds by assuming that the full electron propagator G has a similar representation

$$(G)_{i\sigma,j\sigma} = -\sum_a \frac{1}{\varepsilon_{a\sigma}}\, \varphi_{a\sigma}(\mathbf{r}_i)\, \varphi_{a\sigma}(\mathbf{r}_j)^*$$

The eigenvectors φ are obtained by solving self-consistently the Dyson equation

$$G^{-1} = G_0^{-1} - \Sigma$$

with the electron self-energy

$$(\Sigma)_{i\sigma,j\sigma} = \mathbf{1}_{ij} \sum_{l\sigma'} v_{\sigma\sigma'}(\mathbf{r}_i - \mathbf{r}_l) \sum_{\substack{\text{filled}\\ \text{bands}}} |\varphi_{a\sigma'}(\mathbf{r}_l)|^2$$
$$- v_{\sigma\sigma}(\mathbf{r}_i - \mathbf{r}_j) \sum_{\substack{\text{filled}\\ \text{bands}}} \varphi_{a\sigma}(\mathbf{r}_i)\, \varphi_{a\sigma}(\mathbf{r}_j)^*$$

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

The condensation of different order parameters can be studied through the matrix elements t_{ij}

$$t_{ij}^{(\sigma)} = \sum_{\substack{\text{filled} \\ \text{bands}}} \varphi_{a\sigma}(\mathbf{r}_i)\, \varphi_{a\sigma}(\mathbf{r}_j)^*$$

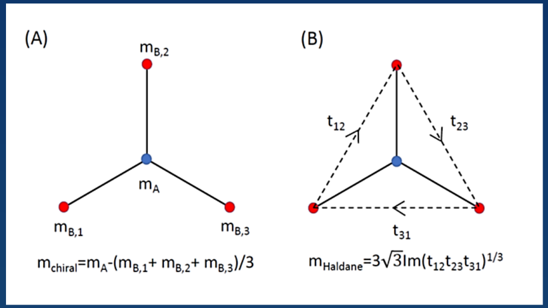
The relevant patterns of symmetry breaking are:

- chiral symmetry breaking characterized by staggered charge order in sublattices A and B

$$C^{(\sigma)} = \sum_{i \in A} t_{ii}^{(\sigma)} - \sum_{i \in B} t_{ii}^{(\sigma)}$$

- time-reversal and valley symmetry breaking with currents circulating along nearest neighbors i_1, i_2, i_3 of each site

$$P_{\pm}^{(\sigma)} = \sum_{i \in A} \mathrm{Im}\,(t_{i_1 i_2}^{(\sigma)} + t_{i_2 i_3}^{(\sigma)} + t_{i_3 i_1}^{(\sigma)}) \pm \sum_{i \in B} \mathrm{Im}\,(t_{i_1 i_2}^{(\sigma)} + t_{i_2 i_3}^{(\sigma)} + t_{i_3 i_1}^{(\sigma)})$$

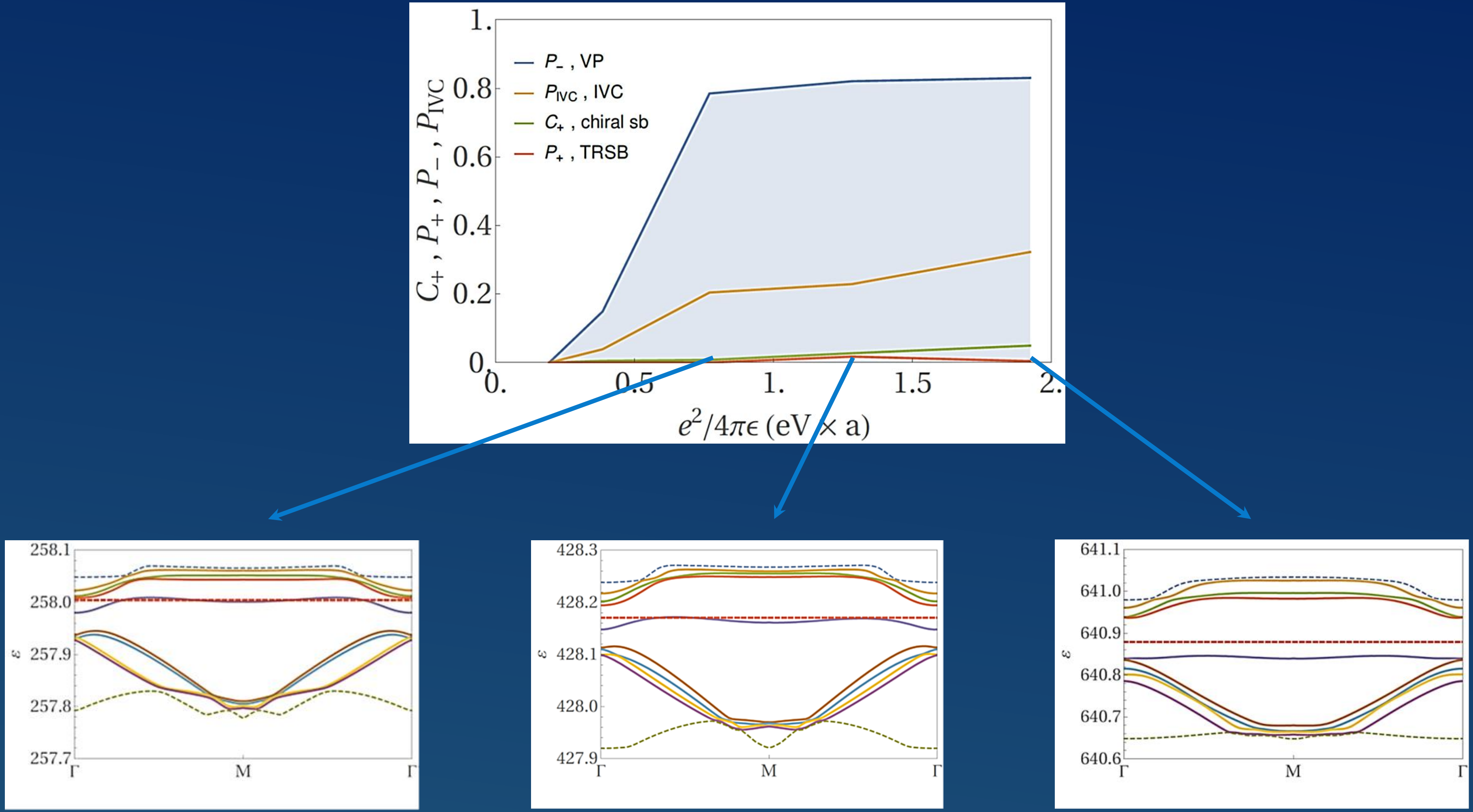


We have to add also the order parameters for intervalley coherence, with currents circulating along the hexagons of the carbon lattice

$$P_{\mathrm{IVC}}^{(\sigma)} = \sum_{\hexagon} n_i\, \mathrm{Im}\,(t_{i_1 i_2}^{(\sigma)} + t_{i_2 i_3}^{(\sigma)} + t_{i_3 i_4}^{(\sigma)} + t_{i_4 i_5}^{(\sigma)} + t_{i_5 i_6}^{(\sigma)} + t_{i_6 i_1}^{(\sigma)})$$

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

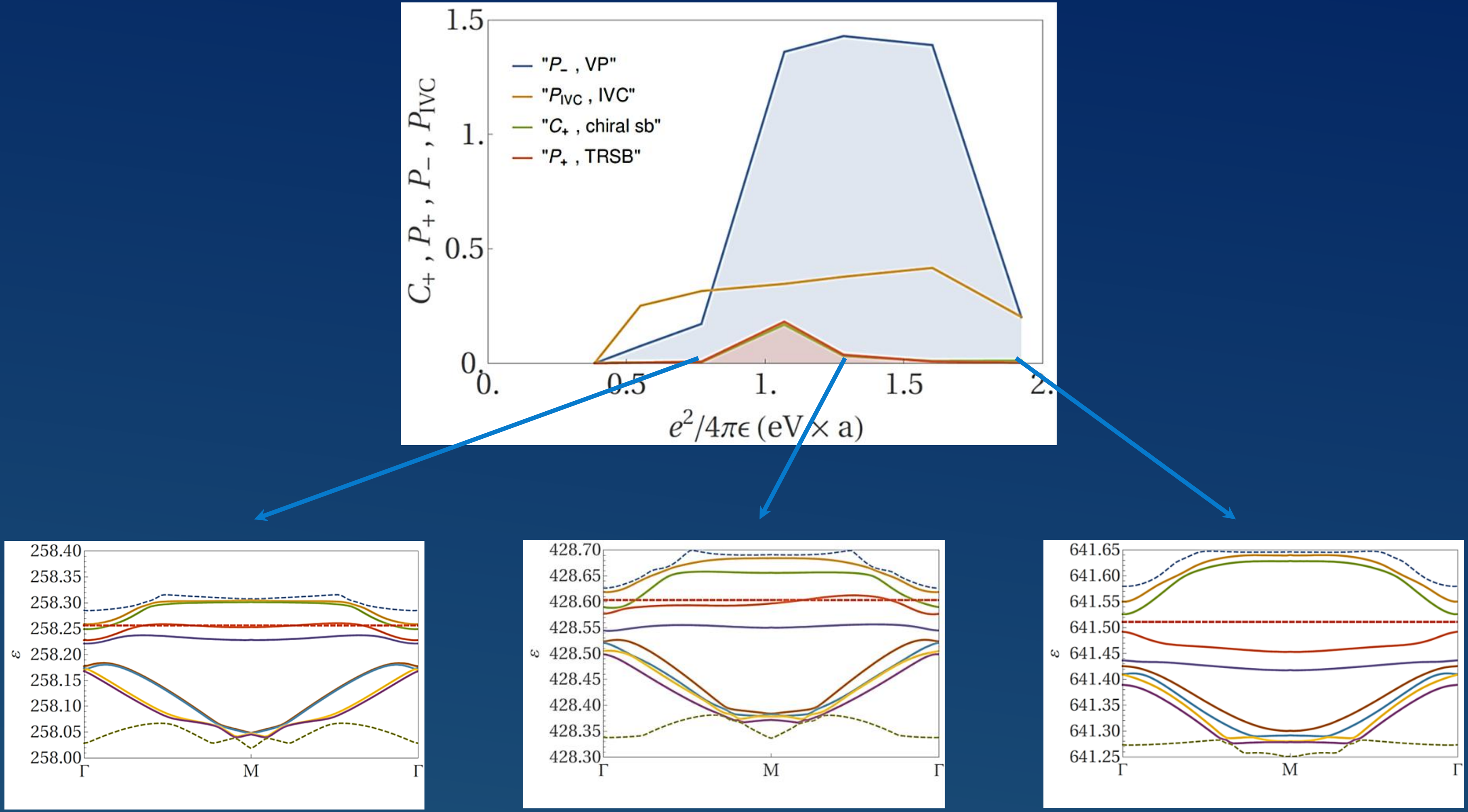
Looking at the phase diagram for filling fraction $n = 2$, we find a valley polarized phase for all interaction strengths



J. G. and T. Stauber, arXiv:2503.05624

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

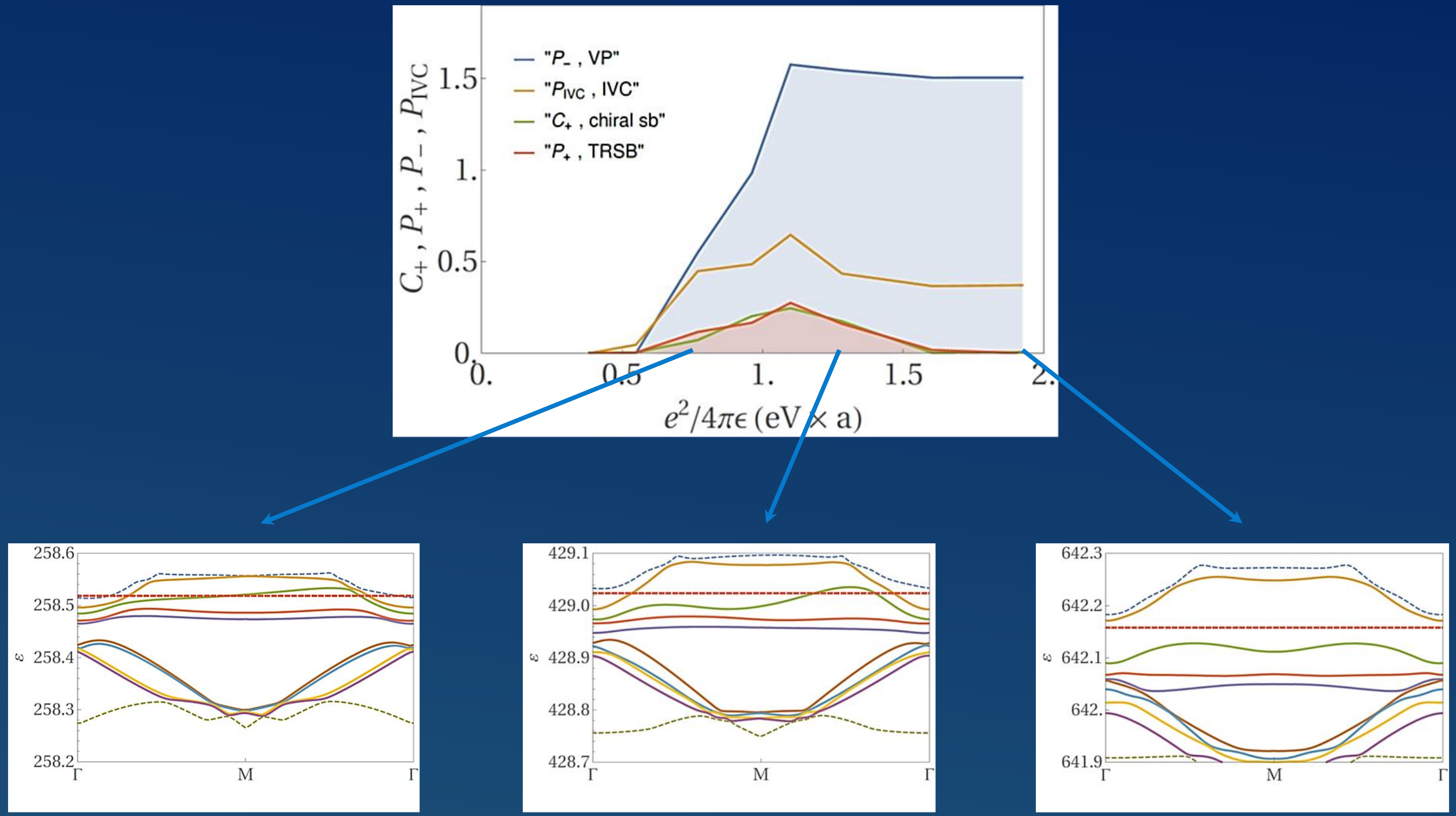
In the phase diagram for $n = 4$, we also find a very strong signal of valley symmetry breaking, with another pattern of intervalley coherence which is in general subdominant



J. G. and T. Stauber, arXiv:2503.05624

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

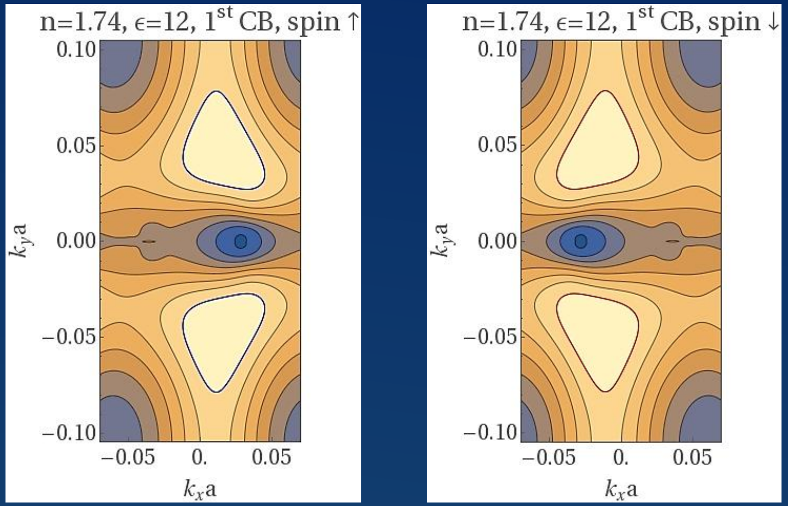
For $n = 6$, we find valley symmetry breaking, but with subdominant patterns of intervalley coherence and time-reversal (parity) symmetry breaking



J. G. and T. Stauber, arXiv:2503.05624

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

The breakdown of valley symmetry has an immediate effect on the pairing instabilities in the bilayer, since they must arise from a valley polarized state with a Fermi line which does not have inversion symmetry



However, we have self-consistent Hartree-Fock solutions in which the two spin projections have opposite sign of the valley polarization, so that inversion symmetry is recovered with the exchange of valley and spin.

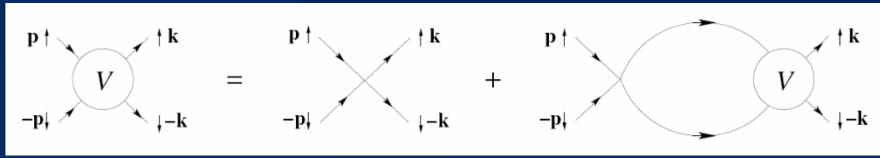
There is then spin-valley locking, which opens the possibility of having Ising superconductivity, with the pairing of electrons arranged so that each spin projection is attached to a different valley.

J. G. and T. Stauber,
Nature Commun. 14, 2746 (2023)

M. Sánchez Sánchez, I. Díaz, J. G.
and T. Stauber, PRL 133, 266603 (2024)

THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

When we have a highly anisotropic Fermi surface, pairing instabilities may arise. we have to look at the divergences in the Cooper-pair (BCS) channel



$$V(\theta,\theta';\omega) = V_0(\theta,\theta') - \frac{1}{(2\pi)^2}\int_0^{\Lambda} d\varepsilon \int_0^{2\pi} d\theta'' \frac{\partial k_\perp}{\partial \varepsilon}\frac{\partial k_\parallel}{\partial \theta''} V_0(\theta,\theta'') \frac{1}{\varepsilon - \frac{\omega}{2}} V(\theta'',\theta';\omega)$$

The self-consistent equation of the BCS vertex can be simplified by reabsorbing the density of states

$$\hat{V}(\theta,\theta';\omega) = \sqrt{\frac{1}{2\pi}\frac{\partial k_\perp(\theta)}{\partial \varepsilon}\frac{\partial k_\parallel(\theta)}{\partial \theta}}\sqrt{\frac{1}{2\pi}\frac{\partial k_\perp(\theta')}{\partial \varepsilon}\frac{\partial k_\parallel(\theta')}{\partial \theta'}}\, V(\theta,\theta';\omega)$$

Taking the derivative with respect to the cutoff Λ, we arrive at

$$\Lambda \frac{\partial}{\partial \Lambda}\hat{V}(\theta,\theta') = \frac{1}{2\pi}\int_0^{2\pi} d\theta''\, \hat{V}(\theta,\theta'')\,\hat{V}(\theta'',\theta')$$

This equation has a divergent flow when any of the harmonics has a coefficient $\hat{V}_n < 0$ at Λ_0

$$\hat{V}_n(\omega) = \frac{\hat{V}_n(\Lambda_0)}{1 + \hat{V}_n(\Lambda_0)\,\log\left(\frac{\Lambda_0}{\omega}\right)}$$

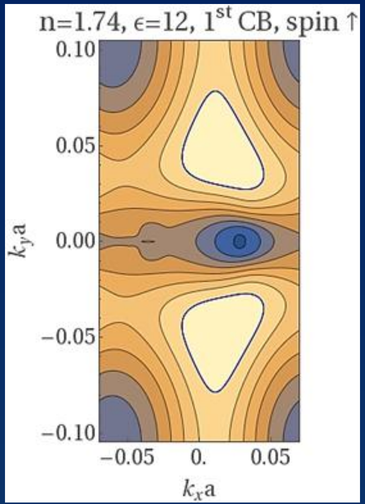
which leads to the signature of the pairing instability in the low-energy limit $\omega \to 0$.

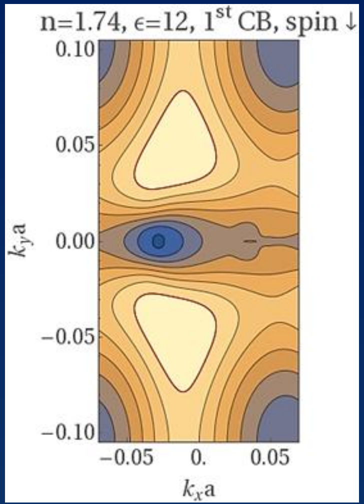
THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

At filling fraction $n \approx 2$, a pairing instability arises from the strong modulation of the electron scattering along the Fermi line.

The bare BCS vertex at the high-energy cutoff Λ_0 can be expressed in the RPA as an iteration of the particle-hole susceptibility χ_q

$$V_0(\phi, \phi') = v_{\boldsymbol{k}-\boldsymbol{k}'} + \frac{v_{\boldsymbol{Q}}^2\, \widetilde{\chi}_{\boldsymbol{k}+\boldsymbol{k}'}}{1 - v_{\boldsymbol{Q}}\, \widetilde{\chi}_{\boldsymbol{k}+\boldsymbol{k}'}} \,,$$





From the expansion of $\hat{V}_0(\phi, \phi')$ in harmonics $\cos(n\phi)$, $\sin(n\phi)$, we look for channels of attraction characterized by coefficients $\hat{V}_n < 0$.

Then, the solution of the scaling equation

$$\hat{V}_n(\omega) = \frac{\hat{V}_n}{1 + \hat{V}_n \log\left(\frac{\Lambda_0}{\omega}\right)}$$

Eigenvalue λ	harmonics
1.28	1
0.41	$\sin(\phi)$
−0.21	$\sin(3\phi)$
0.10	$\sin(4\phi)$
0.09	$\cos(\phi)$
0.06	$\cos(3\phi)$

has a singularity at the energy scale

$$\omega_c = \Lambda_0 \exp\left(-1/\left|\hat{V}_n\right|\right)$$

which is consistent with a critical temperature of ~ 1 K.

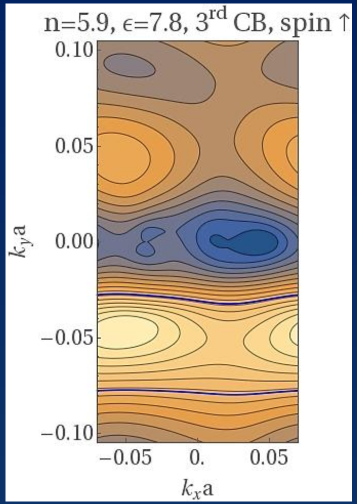
J. G. and T. Stauber, arXiv:2503.05624

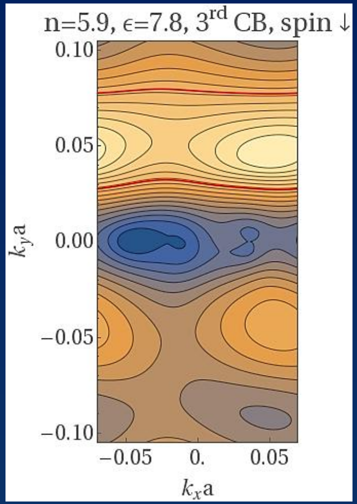
THE ROUTE OF SHEAR TO ISING SUPERCONDUCTIVITY IN BILAYER GRAPHENE

At filling fraction $n \approx 6$, a pairing instability arises from the approximate nesting and enhanced scattering between segments of the Fermi line.

The bare BCS vertex at the high-energy cutoff Λ_0 can be expressed again in the RPA as an iteration of the particle-hole susceptibility χ_q

$$V_0(\phi, \phi') = v_{\boldsymbol{k}-\boldsymbol{k}'} + \frac{v_{\boldsymbol{Q}}^2 \, \widetilde{\chi}_{\boldsymbol{k}+\boldsymbol{k}'}}{1 - v_{\boldsymbol{Q}} \, \widetilde{\chi}_{\boldsymbol{k}+\boldsymbol{k}'}} \, ,$$





From the expansion of $\hat{V}_0(\phi, \phi')$ in harmonics $\cos(n\phi)$, $\sin(n\phi)$, we look for channels of attraction characterized by coefficients $\hat{V}_n < 0$.

Then, the solution of the scaling equation

$$\hat{V}_n(\omega) = \frac{\hat{V}_n}{1 + \hat{V}_n \log\left(\frac{\Lambda_0}{\omega}\right)}$$

has a singularity at the energy scale

$$\omega_c = \Lambda_0 \exp\left(-1/\left|\hat{V}_n\right|\right)$$

which is consistent with a critical temperature of ~ 1 K.

Eigenvalue λ	harmonics
0.86	1
−0.25	$\sin(\phi)$
0.22	$\cos(\phi)$
−0.15	$\cos(2\phi)$
0.12	$\sin(2\phi)$
−0.09	$\cos(3\phi)$

J. G. and T. Stauber, arXiv:2503.05624

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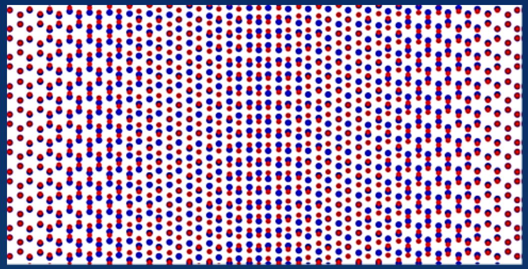
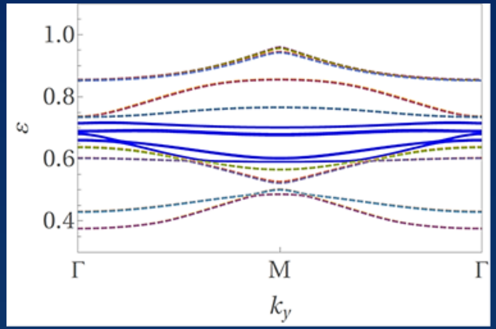
In conclusion,

- we have shown that graphene bilayers with shear provide a suitable setup to study electron correlations in flat low-energy bands about charge neutrality





- the Coulomb interaction forces the breakdown of valley symmetry and parity, inducing highly anisotropic Fermi lines which are prone to develop pairing instabilities