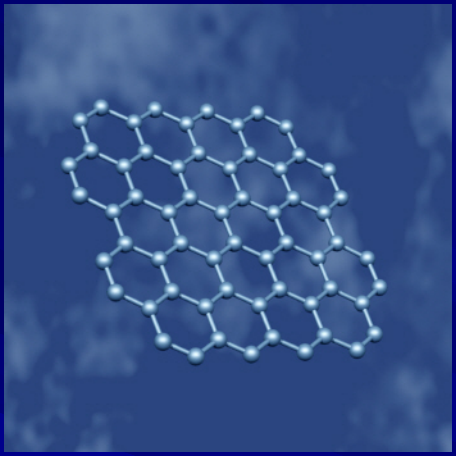
CHARGE POLARIZATION AND PHONON RENORMALIZATION IN GRAPHENE



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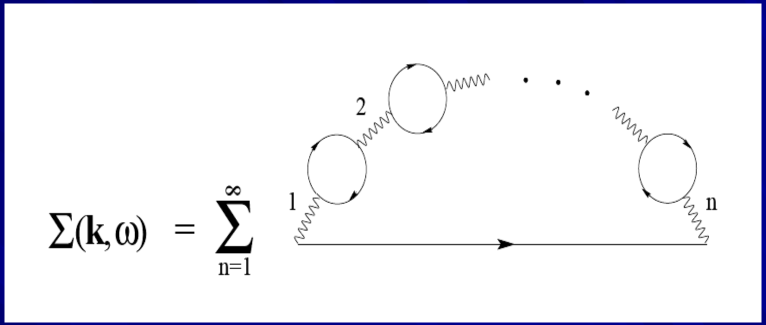
RENORMALIZATION OF QUASIPARTICLES IN GRAPHENE

Many-body effects are very important in graphene as they renormalize significantly the electronic properties.

Computing the electron self-energy with the dressed Coulomb interaction

$$V(\mathbf{q},\omega) = \frac{e^2}{2\kappa|\mathbf{q}| + \frac{e^2}{8}\frac{\mathbf{q}^2}{\sqrt{v_F^2\mathbf{q}^2 - \omega^2}}}$$



we get the renormalization of the electron propagator:

$$\frac{1}{G} = \frac{1}{G_0} - \Sigma$$

$$\approx \omega_k - v_F \boldsymbol{\sigma}\cdot\mathbf{k}$$

$$+ (\omega_k - v_F \boldsymbol{\sigma}\cdot\mathbf{k})\,\gamma(g)\log(E_c/\omega_k) - v_F\boldsymbol{\sigma}\cdot\mathbf{k}\,\beta(g)\log(E_c/\omega_k)$$

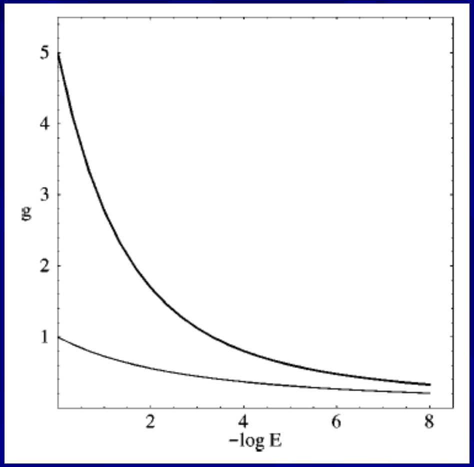
with $g \equiv e^2/16\kappa v_F$

(J. G., F. Guinea and M. A. H. Vozmediano, Phys. Rev. B 59, 2474 (1999))

RENORMALIZATION OF QUASIPARTICLES IN GRAPHENE

To lowest order in perturbation theory, we get

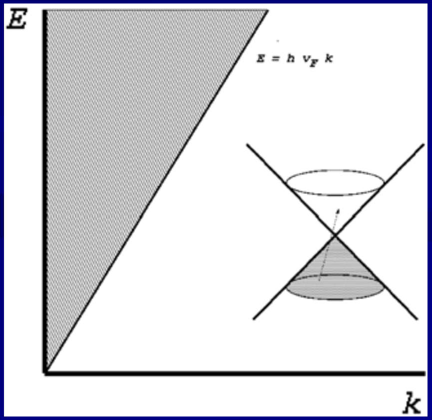
$$\mathrm{Re}\,\Sigma(\mathbf{k},\omega) \propto \left(\frac{e^2}{\kappa v_F}\right)^2 \omega \log(\omega) \propto \frac{\omega}{\log(\omega)}$$



which is consistent with an imaginary part

$$\mathrm{Im}\,\Sigma(\mathbf{k},\omega) \propto \left(\frac{e^2}{\kappa v_F}\right)^2 \omega \propto \frac{\omega}{\log^2(\omega)}$$

However, this does not mean that the quasiparticle decay rate is linear in energy, as the particle-hole excitations are actually confined to the range $\omega_q \geq v_F |\mathbf{q}|$



This is reflected in the singular behavior

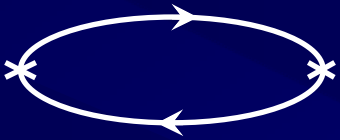
$$\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}| + \varepsilon) \propto v_F|\mathbf{k}|$$

$$\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}| - \varepsilon) = 0$$

(J. G., F. Guinea and
M. A. H. Vozmediano,
Phys. Rev. Lett. **77**, 3589 (1996))

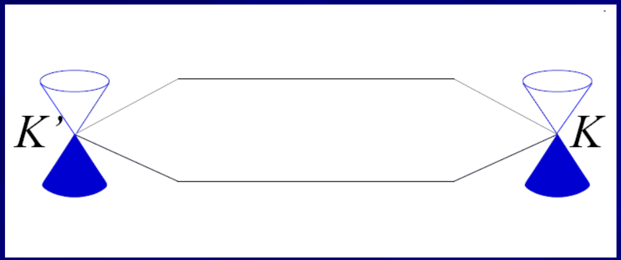
PARTICLE-HOLE SUSCEPTIBILITIES

The role of the different excitations can be analyzed in terms of the susceptibilities:



$$\Pi_0^{(a,b)}(\mathbf{q}, i\varpi_q) = \mathrm{Tr}\int d^2k \int d\varpi_k \, \Gamma \frac{1}{i\varpi_k + i\varpi_q - v_F \boldsymbol{\gamma}^{(a)} \cdot (\mathbf{k}+\mathbf{q})} \Gamma \frac{1}{i\varpi_k - v_F \boldsymbol{\gamma}^{(b)} \cdot \mathbf{k}}$$

$$\begin{pmatrix} 0 & -k_x - ik_y \\ -k_x + ik_y & 0 \end{pmatrix}$$



$$\begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}$$

intravalley polarization

$$\Pi_0^{(K,K)}(\mathbf{q}, \omega) = -\frac{\mathbf{q}^2}{8\sqrt{v_F^2 \mathbf{q}^2 - \omega^2}}$$

intervalley polarization

$$\Pi_0^{(K,K')}(\mathbf{q}, \omega) = \frac{\mathbf{q}^2/2 - \omega^2/v_F^2}{8\sqrt{v_F^2 \mathbf{q}^2 - \omega^2}}$$

phonon self-energy

$$\Pi_0^{(K,K)}(\mathbf{q}, \omega) \propto \sqrt{v_F^2 \mathbf{q}^2 - \omega^2}$$

RENORMALIZATION BY INTERVALLEY PROCESSES

If we focus on the renormalization of the interactions at momentum-transfer *K*:

- Coulomb potential

$$V(\vec{K}+\mathbf{q},\omega) \approx \frac{e^2}{2\kappa K - e^2 \dfrac{\mathbf{q}^2/2-\omega^2/v_F^2}{8\sqrt{v_F^2\mathbf{q}^2-\omega^2}}}$$

(no plasmons around the *K* point)

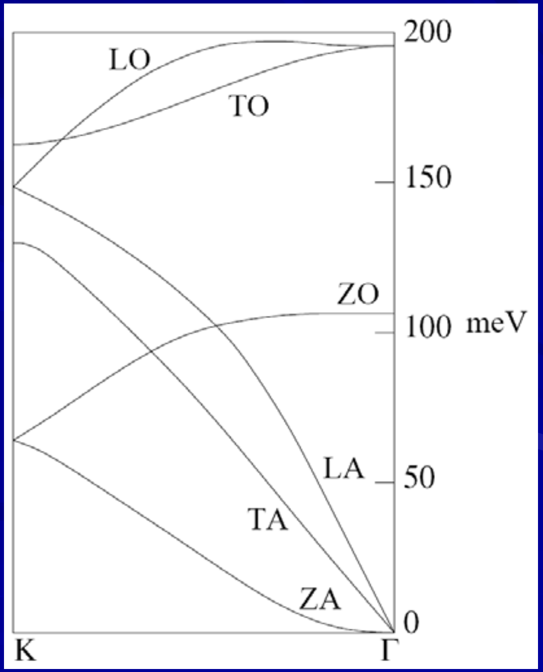
- phonon propagator (optical phonons)

$$D(\mathbf{q},\omega) \approx \frac{2\omega_0}{\omega^2-\omega_0^2+g^2\omega_0\sqrt{v_F^2\mathbf{q}^2-\omega^2}}$$

(Kohn anomalies, A. H. Castro Neto and F. Guinea, Phys. Rev. B **75**, 045404 (2007))

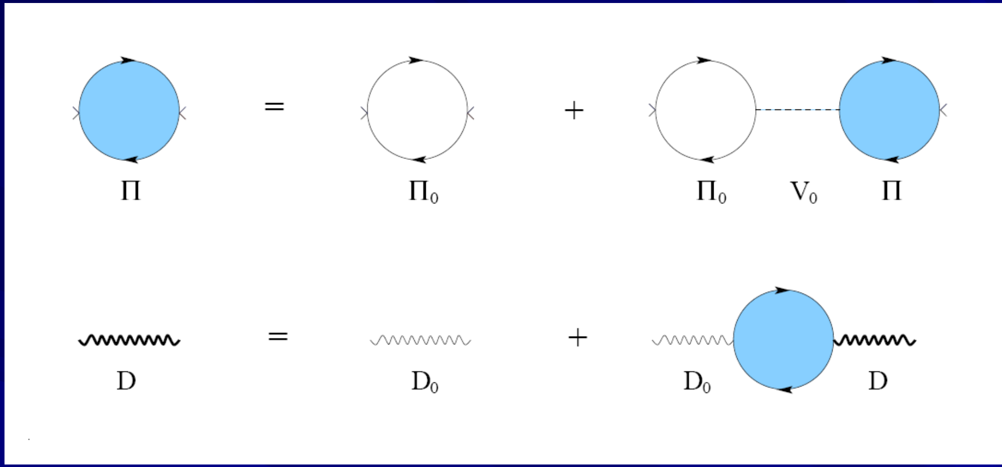
- phonon propagator (out-of-plane phonons)

$$D(\mathbf{q},\omega) \approx \frac{2\omega_0}{\omega^2-\omega_0^2-g^2 2\omega_0 \dfrac{\mathbf{q}^2/2-\omega^2/v_F^2}{8\sqrt{v_F^2\mathbf{q}^2-\omega^2}}}$$



RENORMALIZATION BY INTERVALLEY PROCESSES

However, at this point one has to study the combined effect of the Coulomb and the phonon-mediated interaction:



It turns out that

$$D(\mathbf{q},\omega) \approx \frac{2\omega_0\left(1-(e^2/2\kappa K)\Pi_0^{(K,K')}(\mathbf{q},\omega)\right)}{\omega^2-\omega_0^2+i\varepsilon-\left((\omega^2-\omega_0^2)\dfrac{e^2}{2\kappa K}+2\omega_0 g^2\right)\Pi_0^{(K,K')}(\mathbf{q},\omega)}$$

We have to compare the strength of the phonon-exchange interaction $g^2/v_F^2 \sim 0.1$ and the strength of the Coulomb interaction $e^2/\kappa v_F \sim 1$. However, the latter enters in the above equation with a relative weight $\omega_0/4v_F K \sim 0.001$, so that it can be safely disregarded.

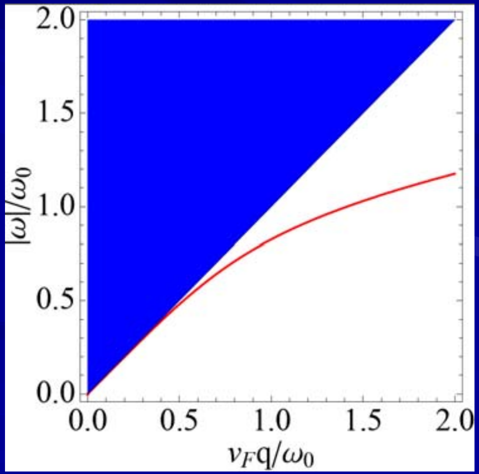
RENORMALIZATION OF PHONONS AT THE K POINT

We can obtain the dispersion of the out-of-plane phonons by looking for the pole of the propagator

$$D(\mathbf{q},\omega) \approx \frac{2\omega_0}{\omega^2-\omega_0^2+i\varepsilon-g^2 2\omega_0 \dfrac{\mathbf{q}^2/2-\omega^2/v_F^2}{8\sqrt{v_F^2\mathbf{q}^2-\omega^2}}}$$

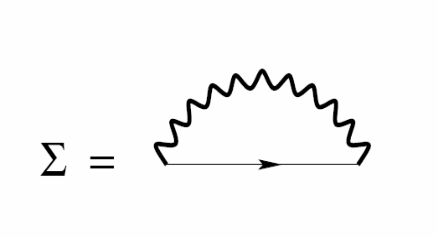
We observe the appearance of very soft phonon modes near the *K* point of graphene, which can be discerned from the particle-hole continuum

$$|\omega_{\mathrm{ph}}(\mathbf{q})| \approx v_F|\mathbf{q}|-\left(\frac{g^2}{8}\right)^2 \frac{|\mathbf{q}|^3}{2v_F\omega_0^2}+\ldots$$



LOW-ENERGY ELECTRONIC PROPERTIES

The existence of soft phonon modes changes significantly the low-energy electronic properties. We have for instance the quasiparticle decay rate



$$\tau^{-1} = -\,\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}|)$$

$$\propto \mathrm{Im}\, i g^2 \int d^2q \int d\omega_q \frac{\omega_k - \omega_q + v_F \boldsymbol{\gamma}^{(a)}\cdot(\mathbf{k}-\mathbf{q})}{-(\omega_k-\omega_q)^2 + (\mathbf{k}-\mathbf{q})^2 - i\varepsilon} D(\mathbf{q},\omega_q)$$

$$\propto g^2 \int_0^{|\mathbf{k}|} dq\, |\mathbf{q}| \int_0^{|\mathbf{q}|} d\Omega_{\mathbf{q}} \left|\frac{\partial\phi}{\partial\Omega_{\mathbf{q}}}\right| \delta\big(Q(\mathbf{q},\Omega_{\mathbf{q}})\big) \quad , \quad \text{where} \quad Q(\mathbf{q},\omega) = \frac{\omega^2-\omega_0^2}{2\omega_0} - g^2 \frac{\mathbf{q}^2/2 - \omega^2/v_F^2}{8\sqrt{v_F^2\mathbf{q}^2 - \omega^2}}$$

We have now quite different behaviors depending on the energy range:

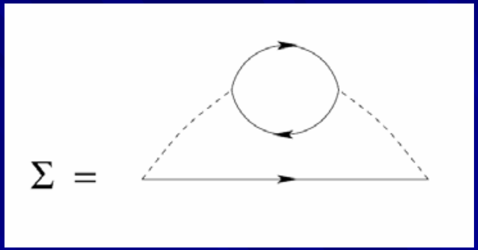
$$\tau^{-1} \propto \begin{cases} \left(\dfrac{g^2}{v_F^2}\right) v_F|\mathbf{k}| & v_F|\mathbf{k}| > \omega_0 \\[2ex] \left(\dfrac{g^2}{v_F^2}\right)^2 \dfrac{v_F^3|\mathbf{k}|^3}{\omega_0^2} & v_F|\mathbf{k}| < \omega_0 \end{cases}$$

(consistent with C.-H. Park *et al.*, Phys. Rev. Lett. **99**, 086804 (2007))

J. G. and E. Perfetto, arXiv:0803.0894

LOW-ENERGY ELECTRONIC PROPERTIES

For comparison, we may derive the quasiparticle decay rate in the case of a screened Coulomb interaction:



$$\tau^{-1} = -\,\mathrm{Im}\,\Sigma(\mathbf{k}, v_F|\mathbf{k}|)$$

$$\propto \mathrm{Im}\, ie^2 \int d^2q \int d\omega_q \frac{\omega_k - \omega_q + v_F \boldsymbol{\gamma}^{(a)} \cdot (\mathbf{k}-\mathbf{q})}{-(\omega_k - \omega_q)^2 + (\mathbf{k}-\mathbf{q})^2 - i\varepsilon} V(\mathbf{q}, \omega_q)$$

$$\propto e^4 \int_0^{|\mathbf{k}|} dq\, |\mathbf{q}| \int_{|\mathbf{q}|}^{|\mathbf{q}|+\delta} d\Omega_{\mathbf{q}} \left| \frac{\partial \phi}{\partial \Omega_{\mathbf{q}}} \right| \frac{1}{\mathbf{q}^2 + \ell^{-2}} \frac{\mathbf{q}^2}{\sqrt{\Omega_{\mathbf{q}}^2 - v_F^2 \mathbf{q}^2}}$$

In the limit $\delta \to 0$, we have a finite quasiparticle decay rate, with two different regimes

$$\tau^{-1} \propto \begin{cases} \left(\dfrac{e^2}{v_F}\right)^2 v_F |\mathbf{k}| & |\mathbf{k}| > \ell^{-1} \\[2ex] \left(\dfrac{e^2}{v_F}\right)^2 v_F \ell^2 |\mathbf{k}|^3 & |\mathbf{k}| < \ell^{-1} \end{cases}$$

(J. G., F. Guinea and M. A. H. Vozmediano, Phys. Rev. Lett. **77**, 3589 (1996), also consistent with E. H. Hwang, B. Y.-K. Hu, and S. Das Sarma, Phys. Rev. B **76**, 115434 (2007))

In conclusion

- in undoped graphene, there is a new branch of soft phonon modes, arising from the renormalization of out-of-plane phonons at the K point
- this new branch modifies the electronic properties at low-energies, leading to a quasiparticle decay rate that goes to zero as $\sim \varepsilon^3$ below the energy scale $\omega_0 \approx 70\,\text{meV}$
- transport properties should be measured at suitably low doping, to look for the expected crossover in the quasiparticle decay rate