CHARGE POLARIZATION AND PHONON RENORMALIZATION IN GRAPHENE

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RENORMALIZATION OF QUASIPARTICLES IN GRAPHENE

Many-body effects are very important in graphene as they renormalize significantly the electronic properties.
Computing the electron self-energy with the dressed Coulomb interaction

\[ V(q, \omega) = \frac{e^2}{2\kappa|q| + \frac{e^2}{8}\frac{q^2}{\sqrt{v_F q^2 - \omega^2}}} \]

we get the renormalization of the electron propagator:

\[ \frac{1}{G} = \frac{1}{G_0} - \Sigma \]

\[ \approx \omega_k - v_F \sigma \cdot k \]

\[ + (\omega_k - v_F \sigma \cdot k) \gamma(g) \log(E_c / \omega_k) - v_F \sigma \cdot k \beta(g) \log(E_c / \omega_k) \]

with \( g \equiv e^2/16\kappa v_F \)

RENORMALIZATION OF QUASIPARTICLES IN GRAPHENE

To lowest order in perturbation theory, we get

\[ \text{Re } \Sigma(k, \omega) \propto \left( \frac{e^2}{\kappa v_F} \right)^2 \omega \log(\omega) \propto \frac{\omega}{\log(\omega)} \]

which is consistent with an imaginary part

\[ \text{Im } \Sigma(k, \omega) \propto \left( \frac{e^2}{\kappa v_F} \right)^2 \omega \propto \frac{\omega}{\log^2(\omega)} \]

However, this does not mean that the quasiparticle decay rate is linear in energy, as the particle-hole excitations are actually confined to the range \( \omega_q \geq v_F|q| \)

This is reflected in the singular behavior

\[ \text{Im } \Sigma(k, v_F|k| + \varepsilon) \propto v_F|k| \]
\[ \text{Im } \Sigma(k, v_F|k| - \varepsilon) = 0 \]

PARTICLE-HOLE SUSCEPTIBILITIES

The role of the different excitations can be analyzed in terms of the susceptibilities:

\[
\Pi_{0}^{(a,b)}(q,i\omega) = \text{Tr} \int d^{2}k d\omega_{k} \Gamma \frac{1}{i\omega_{k} + i\omega_{q} - v_F \gamma^{(a)} \cdot (k + q)} \Gamma \frac{1}{i\omega_{k} - v_F \gamma^{(b)} \cdot k}
\]

\[
\begin{pmatrix}
0 & -k_x - ik_y \\
-k_x + ik_y & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & k_x - ik_y \\
k_x + ik_y & 0
\end{pmatrix}
\]

intravalley polarization

\[
\Pi_{0}^{(K,K)}(q,\omega) = -\frac{q^2}{8\sqrt{v_F^2 q^2 - \omega^2}}
\]

intervalley polarization

\[
\Pi_{0}^{(K,K')} (q,\omega) = \frac{q^2/2 - \omega^2/v_F^2}{8\sqrt{v_F^2 q^2 - \omega^2}}
\]

phonon self-energy

\[
\Pi_{0}^{(K,K)}(q,\omega) \propto \sqrt{v_F^2 q^2 - \omega^2}
\]
RENORMALIZATION BY INTERVALLEY PROCESSES

If we focus on the renormalization of the interactions at momentum-transfer $K$:

- Coulomb potential

\[
V(\vec{K} + \vec{q}, \omega) \approx \frac{e^2}{2\kappa K - e^2 \frac{q^2}{2} - \frac{\omega^2}{v_F^2}}
\]

\[
\approx \frac{e^2}{8\sqrt{v_F^2 q^2 - \omega^2}}
\]

(no plasmons around the $K$ point)

- phonon propagator (optical phonons)

\[
D(q, \omega) \approx \frac{2\omega_0}{\omega^2 - \omega_0^2 + g^2 \omega_0 \sqrt{v_F^2 q^2 - \omega^2}}
\]


- phonon propagator (out-of-plane phonons)

\[
D(q, \omega) \approx \frac{2\omega_0}{\omega^2 - \omega_0^2 - g^2 \omega_0 \frac{q^2}{2} - \frac{\omega^2}{v_F^2}}
\]

\[
\approx \frac{2\omega_0}{8\sqrt{v_F^2 q^2 - \omega^2}}
\]
RENORMALIZATION BY INTERVALLEY PROCESSES

However, at this point one has to study the combined effect of the Coulomb and the phonon-mediated interaction:

\[
\begin{align*}
D(q, \omega) &\approx \frac{2\omega_0 \left( 1 - \frac{e^2}{2\kappa K} \Pi_{0}^{(K,K')}(q, \omega) \right)}{\omega^2 - \omega_0^2 + i \varepsilon - \left( \omega^2 - \omega_0^2 \right) \frac{e^2}{2\kappa K} + 2\omega_0 g^2} \Pi_{0}^{(K,K')} (q, \omega)
\end{align*}
\]

It turns out that

\[
D(q, \omega) \approx \frac{2\omega_0 \left( 1 - \frac{e^2}{2\kappa K} \Pi_{0}^{(K,K')} (q, \omega) \right)}{\omega^2 - \omega_0^2 + i \varepsilon - \left( \omega^2 - \omega_0^2 \right) \frac{e^2}{2\kappa K} + 2\omega_0 g^2} \Pi_{0}^{(K,K')} (q, \omega)
\]

We have to compare the strength of the phonon-exchange interaction \( g^2/v_F^2 \approx 0.1 \) and the strength of the Coulomb interaction \( e^2/\kappa v_F \approx 1 \). However, the latter enters in the above equation with a relative weight \( \omega_0/4v_F K \approx 0.001 \), so that it can be safely disregarded.
RENORMALIZATION OF PHONONS AT THE K POINT

We can obtain the dispersion of the out-of-plane phonons by looking for the pole of the propagator

\[ D(q, \omega) \approx \frac{2\omega_0}{\omega^2 - \omega_0^2 + i\varepsilon - g^2 2\omega_0 \frac{q^2/2 - \omega^2/v_F^2}{8\sqrt{v_F^2 q^2 - \omega^2}}}. \]

We observe the appearance of very soft phonon modes near the K point of graphene, which can be discerned from the particle-hole continuum

\[ |\omega_{ph}(q)| \approx v_F |q| - \left( \frac{g^2}{8} \right)^2 \frac{|q|^3}{2v_F\omega_0^2} + \ldots \]
LOW-ENERGY ELECTRONIC PROPERTIES

The existence of soft phonon modes changes significantly the low-energy electronic properties. We have for instance the quasiparticle decay rate

\[
\tau^{-1} = -\text{Im} \Sigma(k, v_F | k) \quad \propto \text{Im} i g^2 \int d^2 q \int d\omega_q \frac{\omega_k - \omega_q + v_F \gamma^{(a)} \cdot (k - q)}{-(\omega_k - \omega_q)^2 + (k - q)^2 - i \varepsilon} D(q, \omega_q)
\]

\[
\propto g^2 \int_0^{[k]} dq |q| \int_0^{[q]} d\Omega_q \left| \frac{\partial \delta}{\partial \Omega_q} \right| \delta\left(Q(q, \Omega_q)\right), \quad \text{where} \quad Q(q, \omega) = \frac{\omega^2 - \omega_0^2}{2 \omega_0} - g^2 \frac{q^2/2 - \omega^2/v_F^2}{8 \sqrt{v_F^2 q^2 - \omega^2}}
\]

We have now quite different behaviors depending on the energy range:

\[
\tau^{-1} \propto \begin{cases} 
\left(\frac{g^2}{v_F^2}\right) v_F |k| & v_F |k| > \omega_0 \\
\left(\frac{g^2}{v_F^2}\right)^2 \frac{v_F^3 |k|^3}{\omega_0^2} & v_F |k| < \omega_0
\end{cases}
\]


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LOW-ENERGY ELECTRONIC PROPERTIES

For comparison, we may derive the quasiparticle decay rate in the case of a screened Coulomb interaction:

\[ \tau^{-1} = - \text{Im} \Sigma(k, v_F |k|) \]

\[ \propto \text{Im} ie^2 \int d^2 q \int d\omega_q \frac{\omega_k - \omega_q + v_F \gamma^{(a)} \cdot (k - q)}{-(\omega_k - \omega_q)^2 + (k - q)^2 - i\varepsilon} V(q, \omega_q) \]

\[ \propto e^4 \int_0^{|k|} dq q \int_{|q|}^{|q| + \delta} d\Omega_q \left| \frac{\partial \phi}{\partial \Omega_q} \right| \frac{1}{q^2 + \ell^2} \frac{q^2}{\sqrt{\Omega_q^2 - v_F^2 q^2}} \]

In the limit \( \delta \to 0 \), we have a finite quasiparticle decay rate, with two different regimes

\[ \tau^{-1} \propto \begin{cases} \left( \frac{e^2}{v_F} \right)^2 v_F |k| & |k| > \ell^{-1} \\ \left( \frac{e^2}{v_F} \right)^2 v_F \ell^2 |k|^3 & |k| < \ell^{-1} \end{cases} \]

In conclusion

- in undoped graphene, there is a new branch of soft phonon modes, arising from the renormalization of out-of-plane phonons at the K point.
- this new branch modifies the electronic properties at low-energies, leading to a quasiparticle decay rate that goes to zero as $\sim \epsilon^3$ below the energy scale $\omega_0 \approx 70$ meV.
- transport properties should be measured at suitably low doping, to look for the expected crossover in the quasiparticle decay rate.