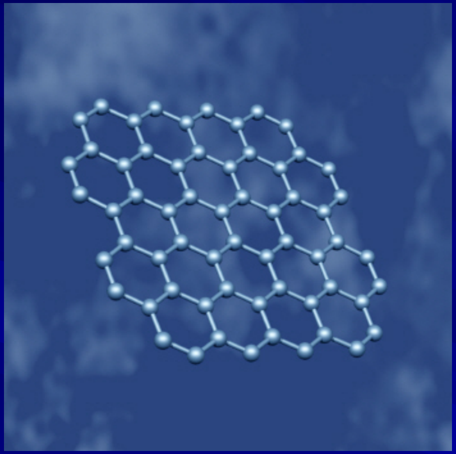
COOPER-PAIR PROPAGATION AND PROXIMITY EFFECT IN GRAPHENE (AND CARBON NANOTUBES)



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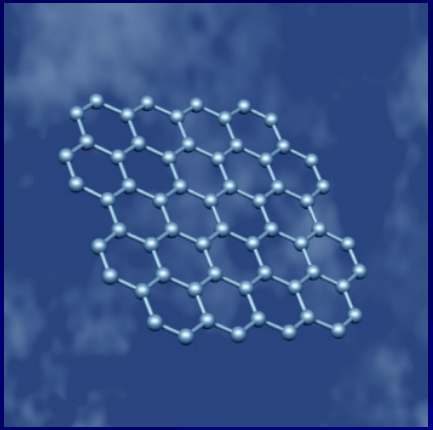
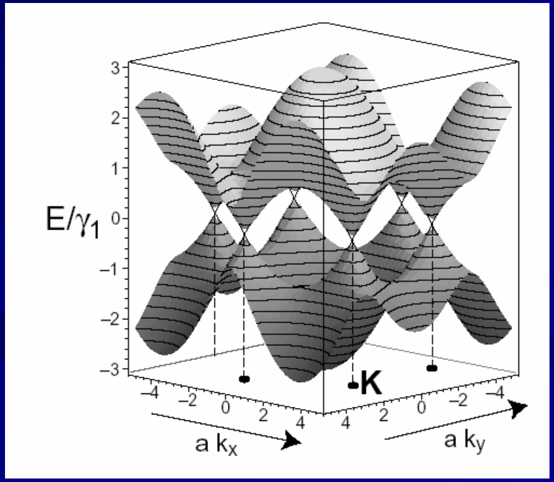
The synthesis of single atomic layers of carbon (graphene) has attracted recently much attention, due to the remarkable properties of the 2D system



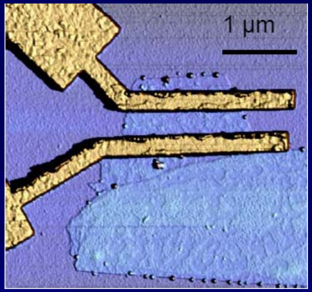
- half-integer quantization of the Hall conductivity
- minimum value of the conductivity, independent of charge concentration
- absence of backscattering from large impurities

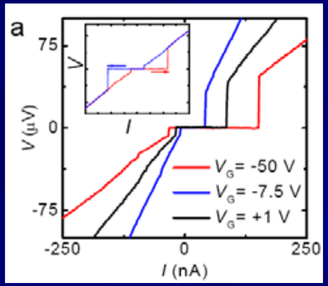


$$H \approx v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}$$

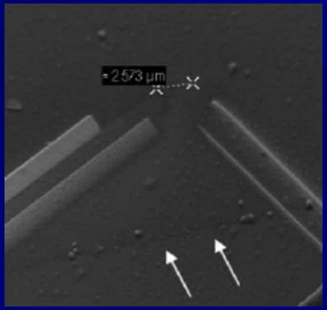
EVIDENCE OF SUPERCONDUCTING CORRELATIONS (PROXIMIT EFFECT) IN GRAPHENE

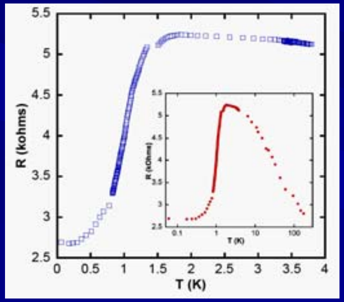
Two different experiments have been carried out in graphene with superconducting electrodes





H. B. Heersche *et al.*, Nature **446**, 56 (2007)



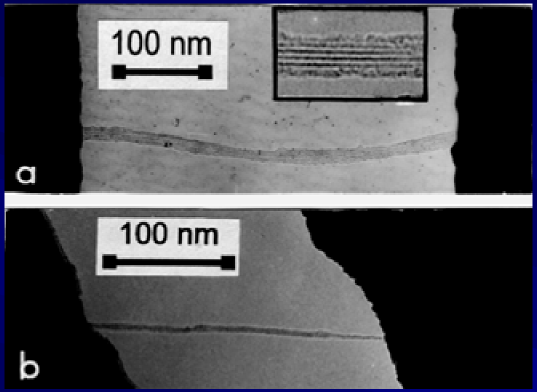


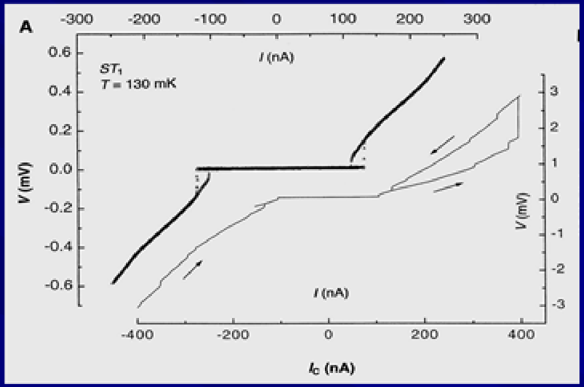
A. Shailos *et al.*, Europhys. Lett. **79**, 57008 (2007)

In the experiment with wide contacts a supercurrent has been measured, while in the opposite case only indirect evidence of superconducting correlations has been obtained

SUPERCURRENTS IN CARBON NANOTUBES

The absence of supercurrents in graphene for long separation L between the contacts may be surprising at first sight, since supercurrents have been already found in ropes of carbon nanotubes with $L \approx 1.7\,\mu\text{m}$





A. Yu. Kasumov *et al.*, Science **284**, 1508 (1999)

Where is the difference between graphene and carbon nanotubes coming from?

SUPERCONDUCTING JUNCTION IN GRAPHENE

We deal with a model of graphene in contact with superconducting electrodes:

y

SC

graphene

SC

$$\left\langle \Psi^{+}_{S1}(\mathbf{r})\Psi^{+}_{S1}(\mathbf{r})\right\rangle \sim e^{-i\chi_1}$$

$$H_0 = v_F \sum_{a=K,K'} \int d^2 r \;\; \Psi^{(a)+}_{\sigma}(\mathbf{r}) \;\; \boldsymbol{\sigma}^{(a)} \cdot \partial \;\; \Psi^{(a)}_{\sigma}(\mathbf{r})$$

$$\left\langle \Psi^{+}_{S2}(\mathbf{r})\Psi^{+}_{S2}(\mathbf{r})\right\rangle \sim e^{-i\chi_2}$$

x

$$t\int_0^W dy \;\; \Psi^{+}_{Sj,\sigma}(x_j, y)\Psi^{(K)}_{\sigma}(x_j, y) \; + \; \mathrm{h.c.}$$

The Josephson current is computed as the derivative of the free energy with respect to the difference χ between the phases of the order parameters in the superconductors:

$$I = 2e \frac{\partial}{\partial \chi} k_B T \log\left(\mathrm{Tr}\, \exp\left(-H / k_B T\right)\right)$$

SUPERCONDUCTING JUNCTION IN GRAPHENE

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$$\left\langle \Psi^+_{S1}(\mathbf{r})\Psi^+_{S1}(\mathbf{r}) \right\rangle \sim e^{-i\chi_1}$$

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$$\left\langle \Psi^+_{S2}(\mathbf{r})\Psi^+_{S2}(\mathbf{r}) \right\rangle \sim e^{-i\chi_2}$$

x

$$t\int_0^W dy \;\; \Psi^+_{Sj,\sigma}(x_j, y)\Psi^{(K)}_{\sigma}(x_j, y) \; + \; \text{h.c.}$$

$$I_J \approx 2e \frac{\partial}{\partial \chi} t^4 \int_0^W dy_1 \int_0^W dy_2 \int_0^W dy_3 \int_0^W dy_4 \int_0^{1/k_B T} d\tau_1 \int_0^{1/k_B T} d\tau_2 \int_0^{1/k_B T} d\tau_3 \int_0^{1/k_B T} d\tau_4$$

$$\Big\langle \Psi_{S1\uparrow}(0, y_1; -i\tau_1)\, \Psi_{S1\downarrow}(0, y_2; -i\tau_2)$$

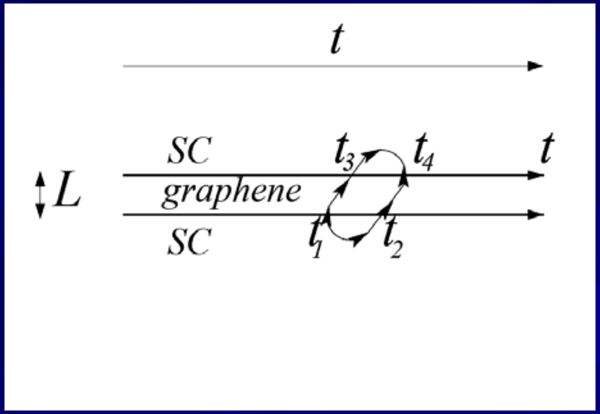
$$\Psi^{(K)+}_{\alpha\uparrow}(0, y_1; -i\tau_1)\, \Psi^{(K')+}_{\alpha\downarrow}(0, y_2; -i\tau_2) \;\; \Psi^{(K)}_{\beta\uparrow}(L, y_3; -i\tau_3)\, \Psi^{(K')}_{\beta\downarrow}(L, y_4; -i\tau_4)$$

$$\Psi^+_{S2\uparrow}(L, y_3; -i\tau_3)\, \Psi^+_{S2\downarrow}(L, y_4; -i\tau_4)\Big\rangle$$

COOPER-PAIR PROPAGATION IN GRAPHENE

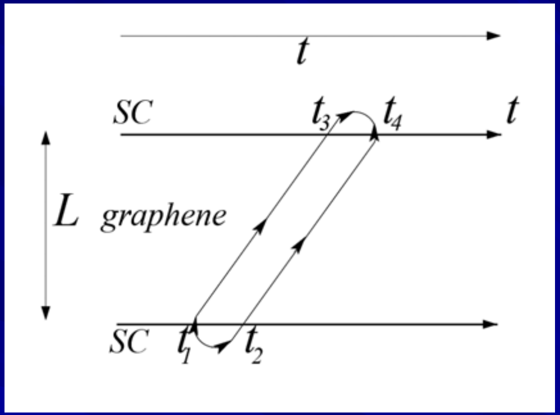
The processes involved are different depending on the relation between the coherence length ξ and the distance L between the electrodes

$L << \xi$



$$\left\langle \Psi^+(t_1)\Psi(t_3)\right\rangle\left\langle \Psi^+(t_2)\Psi(t_4)\right\rangle$$

$L >> \xi$



$$\left\langle \Psi^+(t_1)\Psi^+(t_2)\Psi(t_3)\Psi(t_4)\right\rangle$$

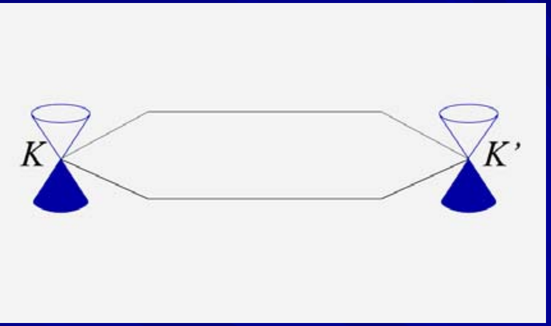
When L is large, the relevant processes are given by the fast tunneling of two electrons and the subsequent propagation in graphene. The supercurrents may be then very sensitive to temperature, interactions, doping (and disorder)

Assuming that $\left\langle \Psi_{Sj}(\mathbf{r},t_1)\Psi_{Sj}(\mathbf{r},t_2)\right\rangle \approx e^{i\chi_j} N\, \delta(t_1 - t_2)$, the supercurrent is given by

$$I_c(T) \approx 2eN^2 t^4 W^2 \int_0^W dy_1 \int_0^W dy_2 \int_0^{1/k_B T} d\tau$$
$$\underbrace{\left\langle \Psi^{(K)+}_{\alpha\uparrow}(0,y_1;0)\, \Psi^{(K')+}_{\alpha\downarrow}(0,y_1;0)\;\; \Psi^{(K)}_{\beta\uparrow}(L,y_2;-i\tau)\, \Psi^{(K')}_{\beta\downarrow}(L,y_2;-i\tau)\right\rangle}_{D(L,\, y_2 - y_1;-i\tau)}$$

In the absence of interaction, the form of the propagator is dictated by the relativistic-like invariance:

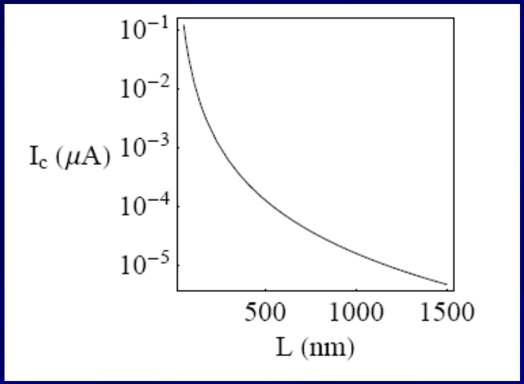
$$D^{(0)}(\mathbf{k},\omega)\Big|_{T=0} = -\frac{1}{8v_F^2}\sqrt{v_F^2\mathbf{k}^2 - \omega^2}$$



This results in a spatial decay of the supercurrent (after subtracting the relative conductances $Nt^2 W / v_F$ at the interfaces):

$$I_c^{(2D)}(T=0) \approx 2eW^2 v_F^2 \int_0^\infty \frac{dk}{2\pi} |\mathbf{k}|\; J_0(|\mathbf{k}|L)\; D^{(0)}(\mathbf{k},0)\; e^{-|\mathbf{k}|/k_c} \;\sim\; ev_F W^2 / L^3$$

The supercurrents suffer a strong decay in graphene over distances of the order of microns

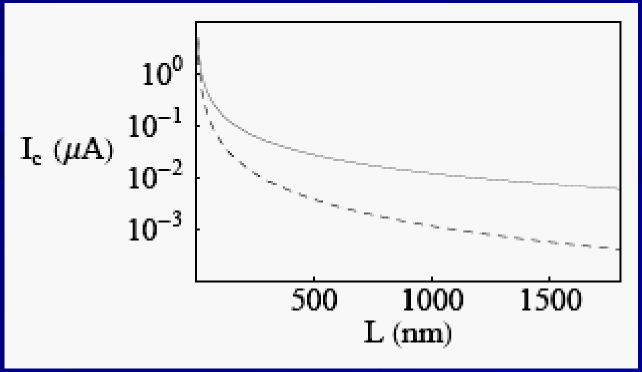


$$W \approx 50\,\mathrm{nm},\ T = 0$$

(J.G. and E. Perfetto, Phys. Rev. B **76**, 155404 (2007); J. Phys.: Condens. Matter **20**, 145218 (2008))

On the other hand, the naïve guess in 1 spatial dimension (carbon nanotubes) would be:

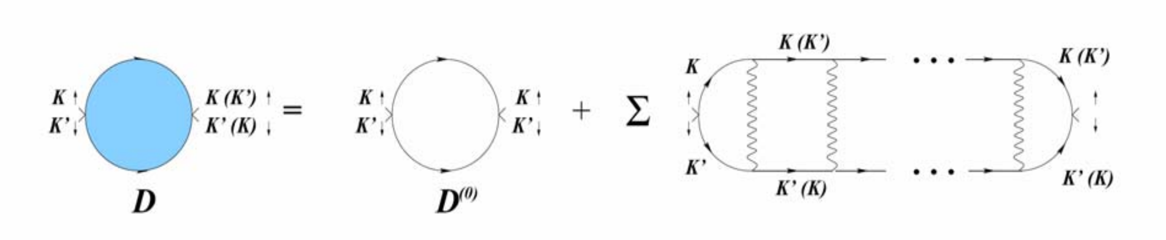
$$I_c^{(1D)}(T=0) \sim ev_F \int_0^\infty \frac{dk}{2\pi} \cos(kL)\, D^{(0)}(k,0)\, e^{-|\mathbf{k}|/k_c} \sim ev_F / L$$



(J. G., Phys. Rev. Lett. **87**, 136401 (2001))

INTERACTION EFFECTS

Anyhow, interactions have to be taken into account. In the case of graphene, we can sum up multiple scattering processes of the electrons in a Cooper pair:



The Cooper pair propagator turns out to be

$$D(\mathbf{k},\omega) \approx \frac{D^{(0)}(\mathbf{k},\omega)}{1+V\,D^{(0)}(\mathbf{k},\omega)}$$

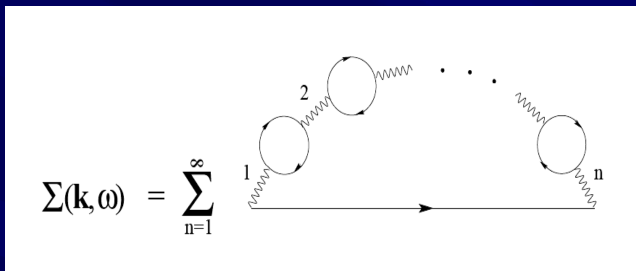
where V stands for the Coulomb potential.

- in the case of inter-valley scattering, the potential is suppressed by the large momentum transfer, $V \sim e^2/2K_F$
- in the case of intra-valley scattering, the product $V\ D^{(0)}$ gives the relative strength $\sim e^2/v_F$

However, one has to check the renormalization of quasiparticle properties at the small energies of the experiments (corresponding to ~ 1 K).

RENORMALIZATION OF QUASIPARTICLES IN GRAPHENE

The quasiparticle properties depend on the energy scale, as the many-body theory requires a low-energy cutoff for its definition

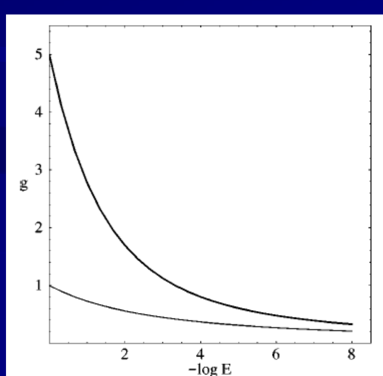


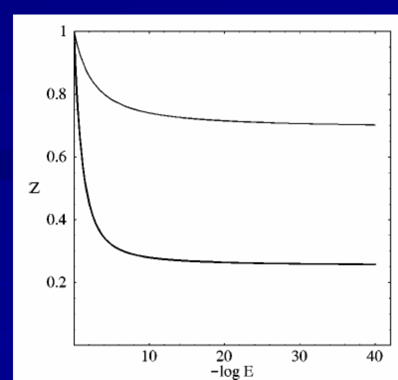
$$\longrightarrow \quad G(\mathbf{k},\omega) = \frac{Z}{\omega - Z_v v_F \boldsymbol{\sigma}\cdot\mathbf{k} + i\Gamma(\omega)}$$

There is a linear quasiparticle decay rate and a logarithmic renormalization of the quasiparticle weight at low energies

$$Z(\omega) \approx 1 - c\left(\frac{e^2}{v_F}\right)^2 \log(E_c/\omega) \quad , \quad \Gamma(\omega) \sim \left(\frac{e^2}{v_F}\right)^2 \omega \qquad\qquad Z_v(\omega) \approx 1 + c'\frac{e^2}{v_F}\log(E_c/\omega)$$

However, the logarithmic renormalization of the Fermi velocity towards increasing values prevents the suppression of the electron quasiparticles





(J. G., F. Guinea and M. A. H. Vozmediano, Phys. Rev. B **59**, 2474 (1999))

INTERACTION EFFECTS IN CARBON NANOTUBES

The Coulomb interaction becomes also irrelevant in large assemblies of carbon nanotubes, described by the coupling of a large number of density fields $\rho^{(a)}(x)$

$$H = \frac{1}{2} v_F \int dk \sum_{a=1}^{N} \rho_r^{(a)}(k) \rho_r^{(a)}(-k) + \frac{1}{2} \int \frac{dk}{2\pi} \left(\sum_{a=1}^{N} \rho_r^{(a)}(k) \sum_{b=1}^{N} V^{(a,b)}(k)\, \rho_r^{(b)}(-k) \right)$$

$$\downarrow$$

$$\frac{1}{2} \int \frac{dk}{2\pi} \left(4 V_C(k) \sum_{a=1}^{N} \rho_+^{(a)}(k) \sum_{b=1}^{N} \rho_+^{(b)}(-k) \right)$$

The Coulomb repulsion operates in a single channel (total charge), and its effects are much suppressed at large *N*. For a superconducting correlation function,

$$D_{SC}(x,0) = C(x,0) \prod_{a=1}^{4N-1} N(x,0) \approx \frac{1}{|x|^{1/2N\mu}} \prod_{a=1}^{4N-1} \frac{1}{|x|^{1/2N}} \,, \qquad \mu = \frac{1}{\sqrt{1 + 4NV_C / \pi v_F}}$$

so that

$$D_{SC}(x,0) \approx \frac{1}{|x|^{2+\gamma}} \qquad\qquad \gamma = \frac{1}{2N} \sqrt{1 + 4NV_C / \pi v_F} - \frac{1}{2N}$$

(J. G., Phys. Rev. Lett. 88, 76403 (2002))

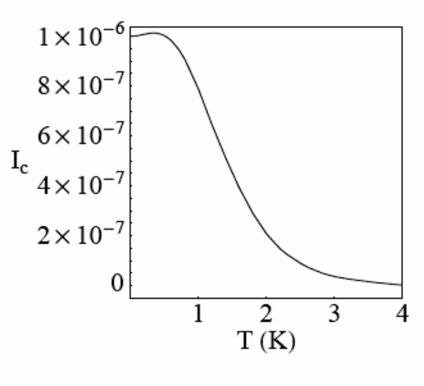
FINITE-TEMPERATURE EFFECTS

The effects of a nonvanishing temperature have a characteristic signature in graphene as well as in carbon nanotubes. In the case of graphene

$$D^{(0)}(\mathbf{k},0) \approx -\frac{|\mathbf{k}|}{8v_F} \qquad k_B T << v_F|\mathbf{k}|$$

$$\approx -\frac{1}{\pi v_F^2}\log(2)\, k_B T - \frac{1}{16\pi}\frac{|\mathbf{k}|^2}{k_B T} \qquad k_B T >> v_F|\mathbf{k}|$$

The crossover gives rise to an abrupt decay of the supercurrent beyond the thermal length $L_T \sim v_F / k_B T$. Conversely, varying T at fixed L there is a crossover temperature at $T^* \sim v_F / k_B L$:



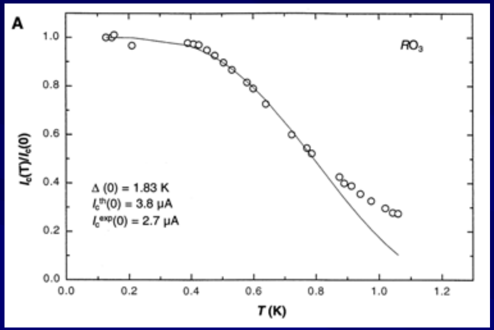
$L \approx 2.5\ \mu\text{m}$ and $W \approx 50$ nm

(J.G. and E. Perfetto, Phys. Rev. B **76,** 155404 (2007); J. Phys.: Condens. Matter **20,** 145218 (2008))

The crossover temperature is found at $T \sim 1$ K for $L \approx 2.5\ \mu\text{m}$, in good correspondence with the temperature of the sharp decrease in the resistance measured by A. Shailos *et al.*

FINITE-TEMPERATURE EFFECTS (CARBON NANOTUBES)

The crossover has been observed in the supercurrents measured in carbon nanotubes, in samples whose length L is larger than the thermal length

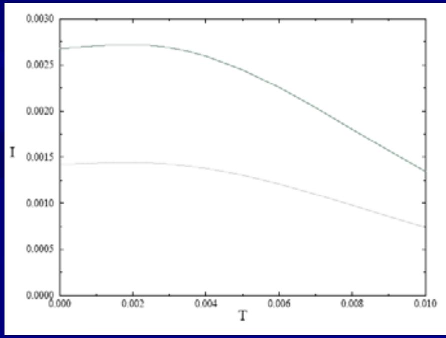


rope with 200 nanotubes

$L \approx 1.7\ \mu m$

(A. Yu. Kasumov *et al.*, Science **284**, 1508 (1999))

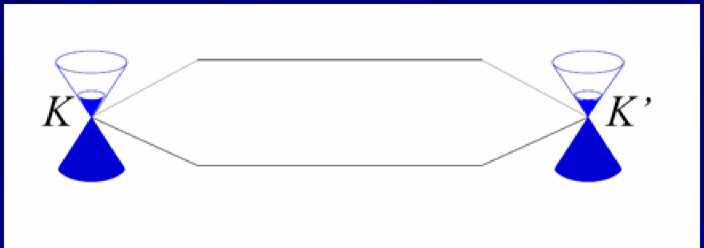
A very flat behavior of the supercurrents has been also found theoretically, below the scale given by the crossover temperature $T^* \sim v_F / k_B L$



(J. G., Phys. Rev. Lett. **88**, 76403 (2002))

DOPING EFFECTS

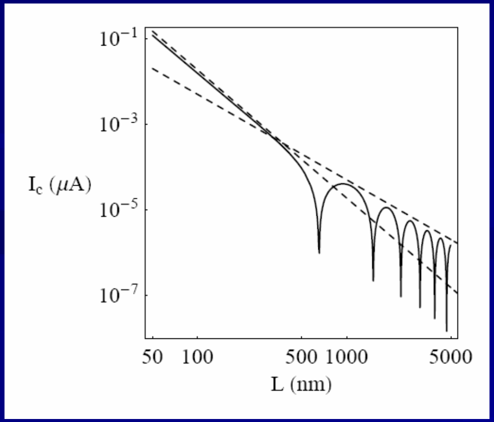
In graphene, the magnitude of the supercurrents can be enhanced by doping the system. Shifting the Fermi level by μ



$$D^{(0)}(\mathbf{k},0) \approx -\frac{1}{2\pi v_F^2}\mu \qquad |\mathbf{k}| < 2\mu$$

$$\approx -\frac{1}{2\pi v_F^2}\mu - \frac{|\mathbf{k}|}{8v_F} + \frac{1}{4\pi v_F}|\mathbf{k}|\arcsin\left(\frac{2\mu}{v_F|\mathbf{k}|}\right) \qquad |\mathbf{k}| > 2\mu$$

This slight change in the infrared behavior is enough to modify the long-distance behavior of the supercurrent to $I_c(0) \sim 1/L^2$



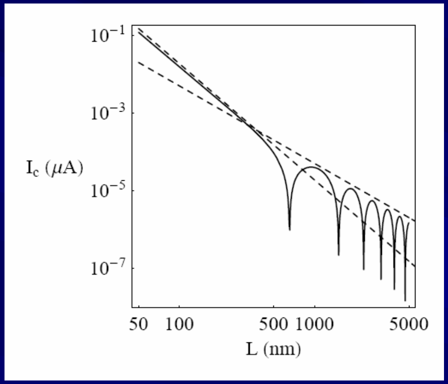
$\mu \approx 1\,\mathrm{meV}$

$W \approx 50\,\mathrm{nm}$

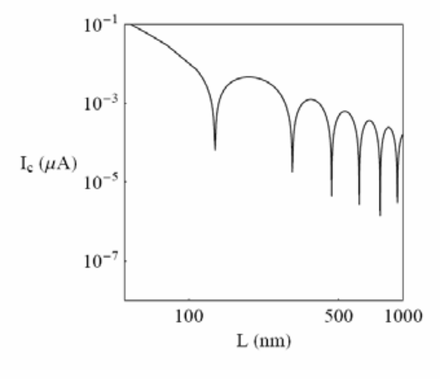
(J.G. and E. Perfetto, Phys. Rev. B **76**, 155404 (2007);
J. Phys.: Condens. Matter **20,** 145218 (2008))

DOPING EFFECTS

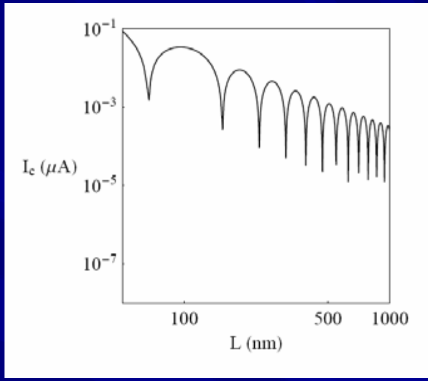
In general, when the Fermi level is shifted by an energy μ , the supercurrents cross over to the smoother $1/L^2$ decay at the length scale $L^* \sim v_F / \mu$



$\mu \approx 1\,\mathrm{meV}$



$\mu \approx 5\,\mathrm{meV}$



$\mu \approx 10\,\mathrm{meV}$

It should be possible then to observe supercurrents of the order of ~ 1 nA (instead of ~ 0.001 nA in undoped graphene) over distances of ~ 1 micron

The investigation of the proximity effect in graphene may be relevant to check that

- interaction effects are not so important at low temperatures (despite the fact that the Coulomb interaction remains unscreened in graphene)
- there is a crossover temperature related to the thermal length beyond which the Cooper pairs are increasingly disrupted
- the magnitude of the supercurrents can be enhanced upon doping, up to measurable values over lengths of the order of ~ 1 micron