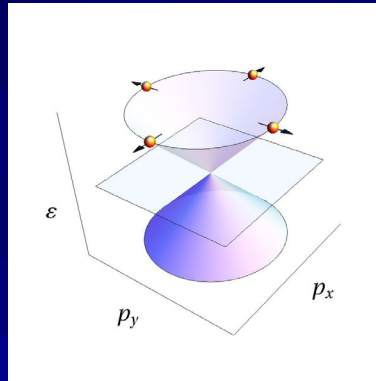


EXCITONIC AND MARGINAL FERMI LIQUID INSTABILITIES IN 2D AND 3D DIRAC SEMIMETALS

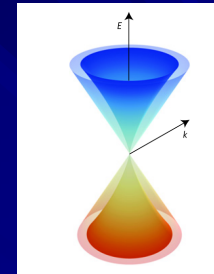
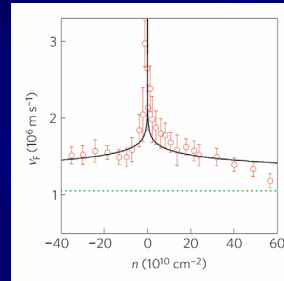
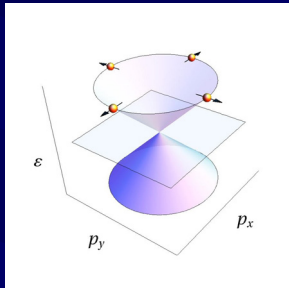


J. González

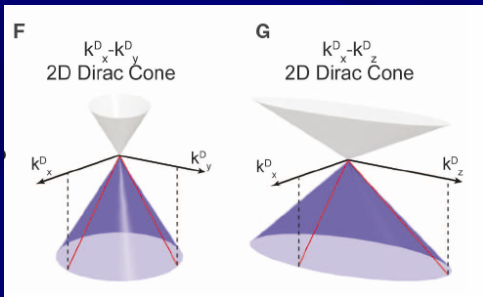
Instituto de Estructura de la Materia, CSIC, Spain

2D AND 3D DIRAC SEMIMETALS

2D and 3D Dirac semimetals offer the possibility to observe the scaling of physical parameters:



D. C. Elias *et al.*, Nature Phys. 7, 701 (2011)



$$e_0^2(\Lambda) = \frac{e^2}{1 - \frac{Ne^2}{6\pi^2 v_F} \log(\Lambda)} \quad ?$$

Z. K. Liu *et al.*, Science **343**, 864 (2014)

See also:

M. Neupane *et al.*, Nature Commun. **5**, 3786 (2014)

S. Borisenko *et al.*, Phys. Rev. Lett. **113**, 027603 (2014)

S.-Y. Xu *et al.*, arXiv:1312.7624.

The question is, are there relevant effects arising from the scaling of these parameters?

2D AND 3D DIRAC SEMIMETALS

A long standing question in graphene is that, focusing on the system with long-range interaction

$$S = \int dt d^2x \Psi_\sigma^+(\mathbf{x}) (i\partial_t - iv_F \gamma_0 \boldsymbol{\gamma} \cdot \partial) \Psi_\sigma(\mathbf{x}) - \frac{e^2}{8\pi} \int dt d^2x d^2x' \Psi_\sigma^+(\mathbf{x}) \Psi_\sigma(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{x}'|} \Psi_{\sigma'}^+(\mathbf{x}') \Psi_{\sigma'}(\mathbf{x}')$$

we should face the strong-coupling regime, with a large relative interaction strength

$$\alpha = \frac{e^2}{4\pi v_F} \approx 2.2 \quad (\text{suspended graphene samples})$$

In these conditions, the electron system is prone to develop an excitonic instability with

$$\langle \Psi^+(\mathbf{r}) \gamma_0 \Psi(\mathbf{r}) \rangle \neq 0$$

$$\int dt d^2x \Psi_\sigma^+(\mathbf{x}) (i\partial_t - iv_F \gamma_0 \boldsymbol{\gamma} \cdot \partial) \Psi_\sigma(\mathbf{x}) \rightarrow \int dt d^2x \Psi_\sigma^+(\mathbf{x}) (i\partial_t - iv_F \gamma_0 \boldsymbol{\gamma} \cdot \partial - m\gamma_0) \Psi_\sigma(\mathbf{x})$$

2D AND 3D DIRAC SEMIMETALS

There have been many analyses of the dynamical generation of a mass in graphene:

- **Gap equation, 1/N approximation.** D. V. Khveshchenko, PRL 87, 246802 (2001);
E. V. Gorbar, V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, PRB 66, 045108 (2002).
- **Gross-Neveu interactions.** I. F. Herbut, V. Juricic and O. Vafek, PRB 80, 075432 (2009);
V. Juricic, I. F. Herbut and G. W. Semenoff, PRB 80, 081405 (2009).
- **Lattice field theory.** J. E. Drut and T. A. Lähde, PRL 102, 026802 (2009); PRB 79, 241405(R) (2009)
 $\Rightarrow$ critical $\alpha_c \approx 1.08$
(see also W. Armour, S. Hands, C. Strouthos, PRB 81, 125105 (2010))
- **Ladder approximation, static polarization.** J. Wang, H. A. Fertig and G. Murthy, PRL 104, 186401 (2010); O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 80, 165429 (2009)
 $\Rightarrow$ critical $\alpha_c \approx 1.62$
- **Gap equation, dynamical screening.** O. V. Gamayun, E. V. Gorbar and V. P. Gusynin, PRB 81, 075429 (2010)
 $\Rightarrow$ critical $\alpha_c \approx 0.92$
- **Effect of Fermi velocity renormalization.** J. Sabio, F. Sols and F. Guinea, PRB 82, 121413(R) (2010); J. G., PRB 85, 085420 (2012)
 $\Rightarrow$ critical $\alpha_c \gtrsim 3.8$
- **Ladder approximation, dynamical screening + electron self-energy corrections.**
J. G., PRB 85, 085420 (2012)
 $\Rightarrow$ critical $\alpha_c \approx 1.75$

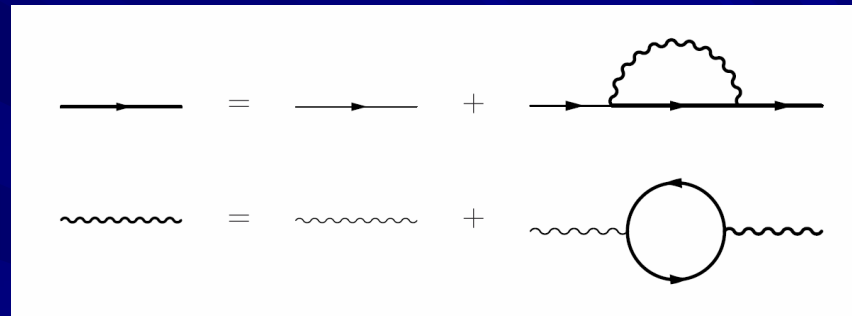
2D AND 3D DIRAC SEMIMETALS

The investigation of the strong-coupling system requires however a nonperturbative approach

Here we propose the resolution of the Schwinger-Dyson equation (bare vertex approximation):

$$G^{-1}(\mathbf{k}, \omega) = \omega - v_F \gamma_0 \boldsymbol{\gamma} \cdot \mathbf{k} - \Sigma(\mathbf{k}, \omega)$$

$$D^{-1}(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{e^2} - \Pi(\mathbf{q}, \omega)$$



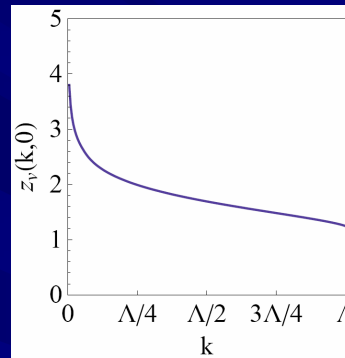
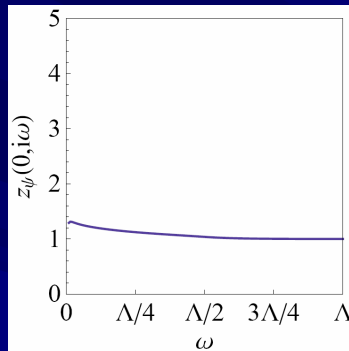
The self-consistent solution for the Dirac fermion propagator

$$G(\mathbf{k}, \omega) = \frac{1}{z_\psi(\mathbf{k}, \omega) \omega - z_v(\mathbf{k}, \omega) v_F \gamma_0 \boldsymbol{\gamma} \cdot \mathbf{k} + z_m(\mathbf{k}, \omega) \gamma_0}$$

encodes then the renormalization of the quasiparticle parameters.

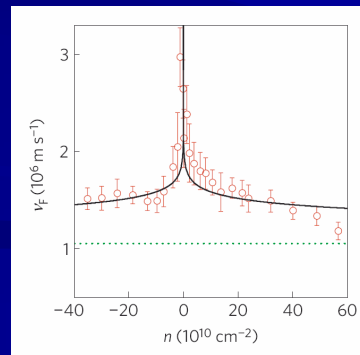
2D DIRAC SEMIMETALS

The self-consistent solution for the fermion propagator shows a renormalization of quasiparticle parameters in which $z_\psi(\mathbf{k},\omega)$ remains bounded while $z_v(\mathbf{k},\omega)$ diverges in the low-energy limit



J.G., arXiv:1502.07640

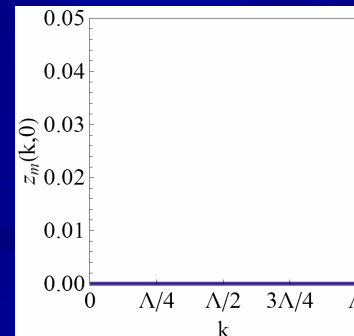
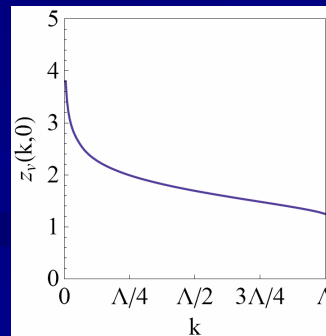
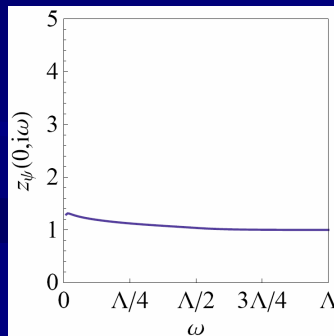
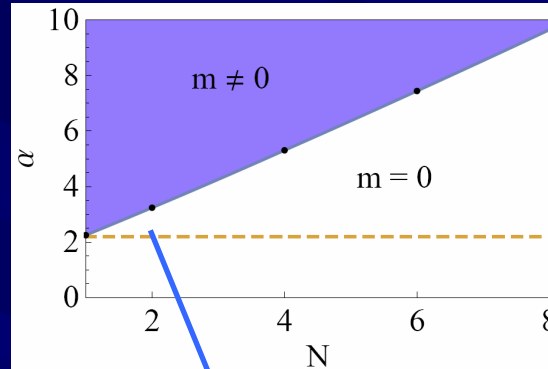
This increase of the Fermi velocity has been observed in graphene at very low doping levels



D. C. Elias *et al.*, Nature
Phys. 7, 701 (2011)

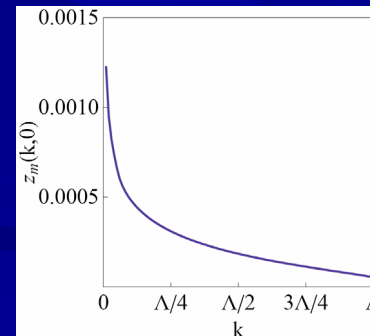
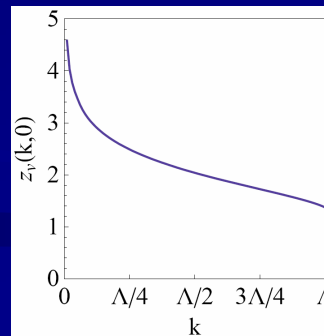
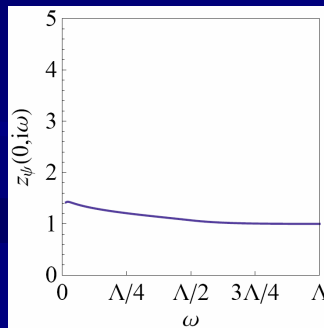
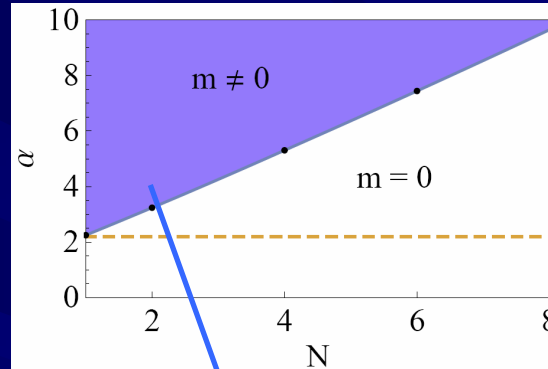
2D DIRAC SEMIMETALS

But in general we may expect two different phases in 2D Dirac semimetals, depending on whether a mass is dynamically generated for the Dirac fermions



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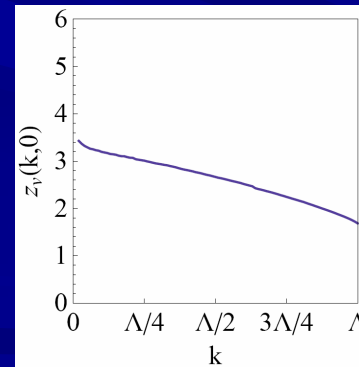
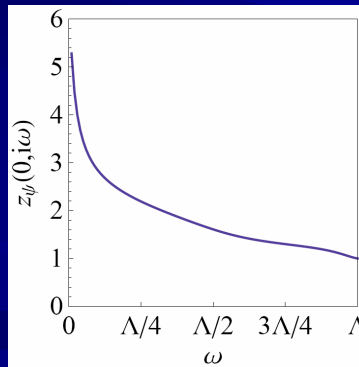
3D DIRAC SEMIMETALS

3D Dirac semimetals have in general a behavior quite different than their 2D analogues

A self-consistent resolution of the Schwinger-Dyson equations is again possible with

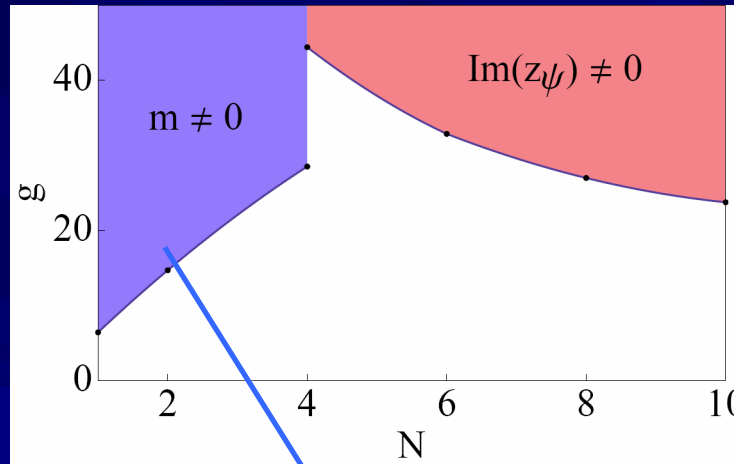
$$G(\mathbf{k}, \omega) = \frac{1}{z_\psi(\mathbf{k}, \omega) \omega - z_v(\mathbf{k}, \omega) v_F \gamma_0 \boldsymbol{\gamma} \cdot \mathbf{k} + z_m(\mathbf{k}, \omega) \gamma_0}$$

But now the most significant feature is the attenuation of electron quasiparticles at low energies

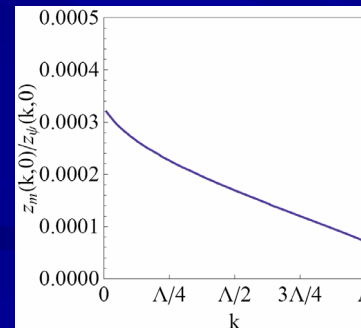
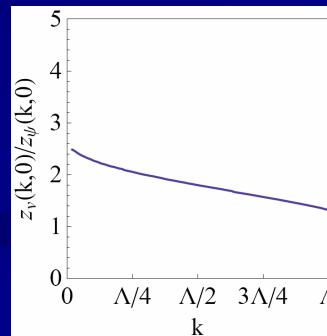
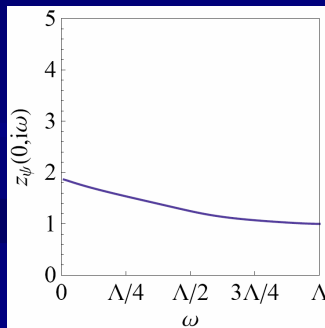


3D DIRAC SEMIMETALS

3D Dirac semimetals have a richer phase diagram depending on the number N of Dirac fermions



$$g \equiv \frac{Ne^2}{4\pi v_F}$$

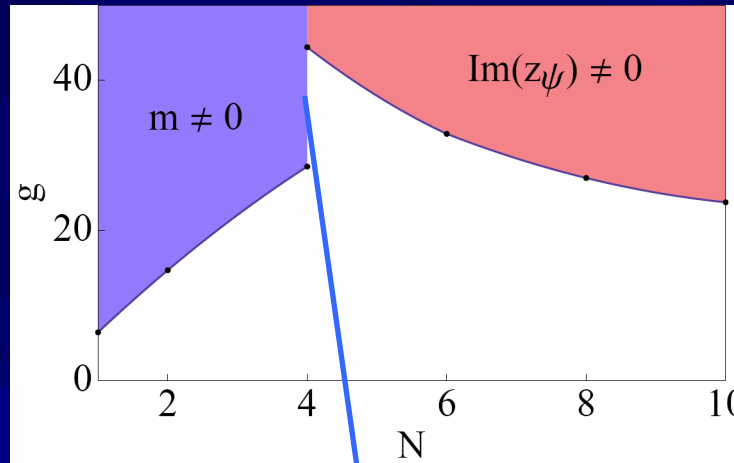


J.G., arXiv:1502.07640

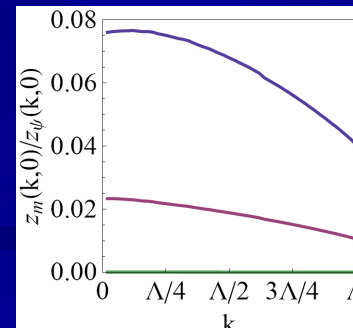
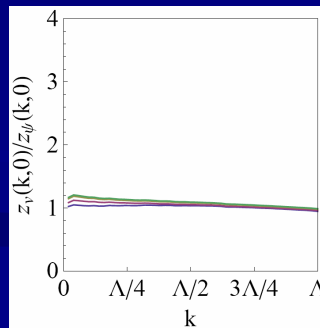
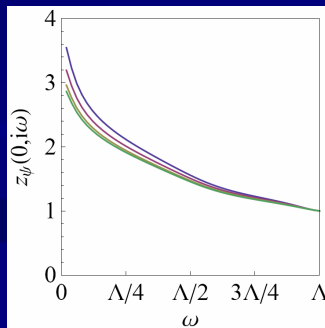
J.G., PRB 90, 121107(R) (2014)

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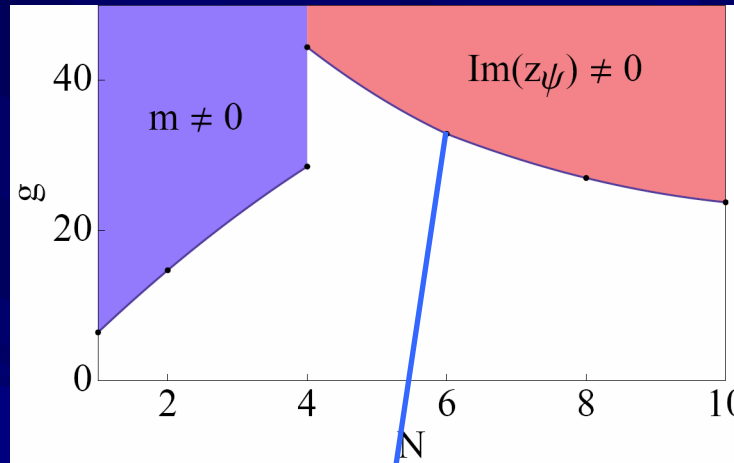


$$g \equiv \frac{Ne^2}{4\pi v_F}$$

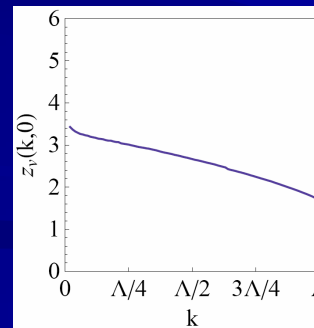
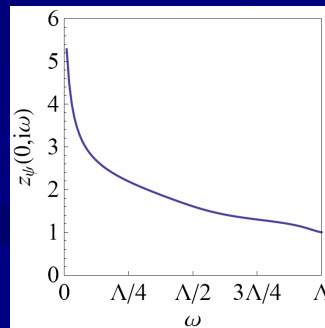


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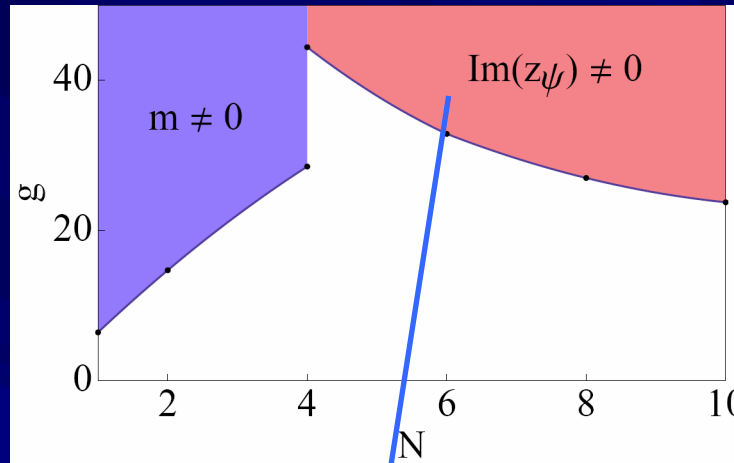


$$g \equiv \frac{Ne^2}{4\pi v_F}$$

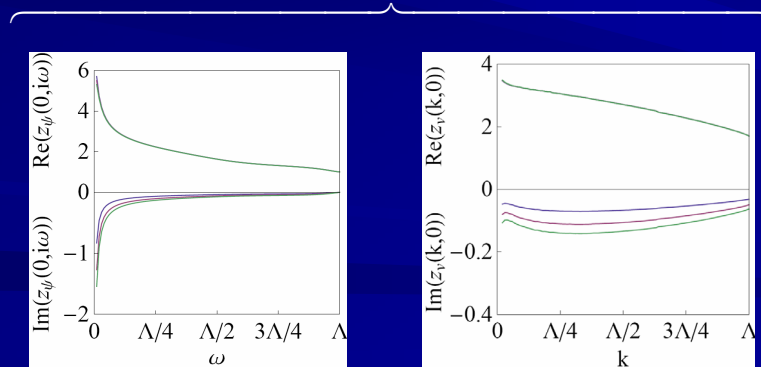


3D DIRAC SEMIMETALS

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J.G., arXiv:1502.07640

J.G., PRB 90, 121107(R) (2014)

3D DIRAC SEMIMETALS

The effect of strong attenuation of electron quasiparticles is also supported by renormalization group analyses in the large- N limit

$$\Sigma(\mathbf{k}, \omega) = \sum_{n=1}^{\infty} \text{Diagram } n$$

$$\frac{1}{G} = Z_{\omega}(\omega - Z_v v_F \gamma_0 \boldsymbol{\gamma} \cdot \mathbf{k}) - \Sigma$$

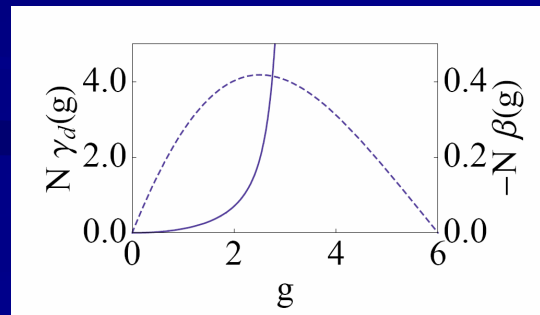
The electron quasiparticles get an anomalous dimension while v_F also scales at low energies

$$G(s\mathbf{k}, s\omega) = s^{-1+\gamma} G(\mathbf{k}, \omega)$$

with

$$\gamma(g) = \frac{\mu}{Z_{\omega}} \frac{\partial Z_{\omega}}{\partial \mu}, \quad \beta(g) = \frac{\mu}{v_F} \frac{\partial v_F}{\partial \mu}, \quad g \equiv \frac{Ne^2}{2\pi^2 v_F}$$

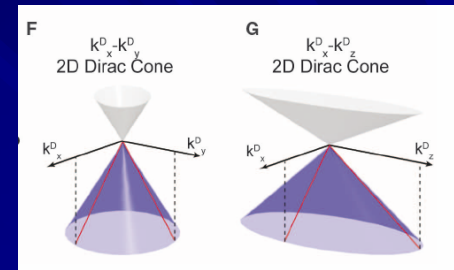
At large N , it is found that $\gamma(g)$ diverges at a critical coupling $g_c = 3$



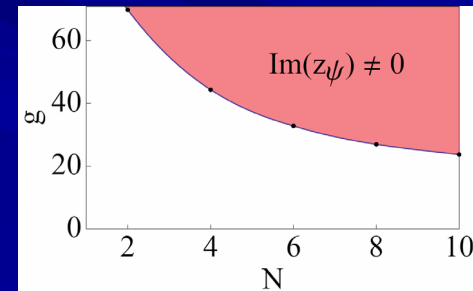
3D DIRAC SEMIMETALS

In conclusion, there should be good prospects to observe these phases in suitable materials

- In the case of known 3D Dirac semimetals, there is a large anisotropy of the electronic dispersion, which may favor the breakdown of the chiral symmetry
- In the case of Weyl semimetals, the dynamical generation of mass is absent since there is only one electron chirality, therefore the only strong coupling phase would correspond to the strongly renormalized electron liquid



Z. K. Liu *et al.*, Science **343**, 864 (2014)



J.G., arXiv:1502.07640